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Research Paper

The simulation and discretisation of random fields for probabilistic finite element analysis of soils using meshes of arbitrary triangular elements

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ABSTRACT

This paper presents a new methodology to discretise random fields for reliability assessment by the Random Finite Element Method. The methodology maps a random field to an arbitrary triangular finite element mesh by local averaging. Derivations for the variance reduction and covariance between local averages of triangular areas using numeric integration are presented. The new methodology is compared against a published reliability assessment of a drained slope. The results matched expectations that not accounting for spatial variability will, in the case analysed, significantly underestimate reliability. A method to generate random fields using a form of Cholesky decomposition appropriate even for singular covariance matrices is presented and analysed. Finally, the derivations for the discretisation of random fields onto triangular meshes are presented for three dimensional tetrahedral elements.

1. Introduction

Soils exhibit significant variations in their measured material property parameters even within apparently homogeneous layers [14]. The typical approach to the modelling of soil property variability in geotechnical engineering involves either the assumption of homogeneity or the application of simple probabilistic models. These models do not adequately quantify the risks of a given design [32]. In this paper, a new development in advanced probabilistic modelling of soils using random fields is presented.

Reliability of a design can be defined in a probabilistic sense as the chance that a structure is able to withstand the loads to which it will be subjected. The majority of simple numerical modelling techniques underestimate the reliability of geotechnical designs provided the factor of safety is greater than one [33]. Excessive conservatism may lead to decisions where either safe designs are considered to have unacceptable levels of risk or, alternatively, over time the perception of what constitutes acceptable risk may become distorted in practice. The cost of overdone conservatism, particularly in long design life civil projects, is economically wasteful and may even render an otherwise sound design financially unviable [32,27]. Of course, a lack of conservatism could result in disaster. When comparing strengths and weaknesses of different numerical analysis models, the fundamental issue is the manner in which a model quantifies risk. Simple analysis methods, such as deterministic limit equilibrium analyses, rely almost entirely on practitioner judgement and qualitative assessment in order to assess risk. Probabilistic methods provide a means to assess the reliability of a given design in terms of acceptable risk. This paper focusses specifically on modelling the variability of soils.

Soil property variability can be modelled as being composed of two components, point and spatial variability. Point variability describes the variations in a measured property recorded independent of position (e.g. direct shear tests carried out on multiple samples taken from across a site) typically modelled with a probability density function. Spatial variability describes how the value of some parameter varies with distance between points and can be defined by a correlation function. The mathematical construct that combines point and spatial variability is termed a random field. Random field models of soil parameters have become increasingly common as the reliability of a design is known to be sensitive to the effects of spatial variability [13,28].

Several methods exist for the computation of the reliability of a given design. One of the most promising techniques available is the Random Finite Element Method (RFEM) [14]. In this methodology, a Monte Carlo Simulation is undertaken in which a random field is repeatedly simulated, mapped onto a finite element mesh and then solved deterministically by finite elements for failure/non-failure. The key advantage of this approach is that the full system

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reliability is computed, as opposed to methods which only analyse a single failure mechanism. Assuming enough simulation runs are completed, the stresses within the finite element model are able to seek out the probabilistic critical failure mechanism.

RFEM makes use of local averaging theory in order to map a random field over an infinite set of points to a discrete finite element mesh [14]. The use of local averaging for random field discretisation is well supported [14,43,44]. A disadvantage with RFEM, as it has been implemented in previous studies, is that only rectangular finite element meshes have been used. The local average subdivision (LAS) algorithm, used heavily in [14], would be difficult to carry out over arbitrarily oriented elements. Regular, rectangular meshes can be impractical. Arbitrarily oriented triangular elements may be required to mesh a problem domain when complicated geometry is present. In this paper, a methodology for the simulation and discretisation of random fields, including the effects of local averaging, for finite element meshes of triangular elements is presented. This methodology was implemented in a computer program, NIRFS, developed by the authors.

In order to test the validity of the random field simulation methodology presented, results from NIRFS were compared against a published reliability assessment of a slope stability problem. Many studies have focussed on probabilistic slope stability analysis, [14] presents a list of 21 significant references. Ji et al. in [27] present the results of a reliability assessment of a simple drained slope based on a limit equilibrium approach where the material properties along the critical failure surface are modelled using random fields. A comparison between NIRFS and the analysis in [27] is presented in Section 4 of this paper.

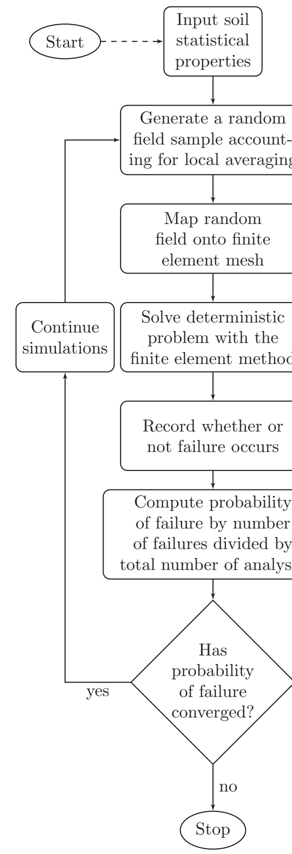

Fig. 1. Flowchart of RFEM simulation methodology.

2. Computing the correlation structure of locally averaged random fields

2.1. Monte Carlo Simulation by the Random Finite Element Method

The Random Finite Element Method [14,20] can be used to compute the system reliability of design. RFEM is a Monte Carlo Simulation process. For each discrete simulation, a random field is simulated, mapped onto a finite element mesh using local averaging and then solved deterministically by finite elements for failure/non-failure. The probability of failure can then be computed as the proportion of failed simulations to the total number of simulations. A flowchart of this procedure is presented in Fig. 1.

2.2. Random field representation

Extensive formal definitions of random fields can be found in [14,43,1]. Gaussian random fields are commonly used as they are completely specified by their first two moments, the mean and covariance of the random field. A Gaussian random field is represented by an infinite multivariate Gaussian distribution between all points, $(x_1, x_2, \ldots, x_k)$, in some spatial domain. Mathematically, this is given as:

$$f_{x_1,x_2,\ldots,x_k}(x_1, x_2, \ldots, x_k) = \frac{1}{(2\pi)^{\frac{k}{2}}|\mathbf{C}|^{\frac{1}{2}}} \exp\left(-\frac{(x-\mu)^T\mathbf{C}^{-1}(x-\mu)}{2}\right) \quad (1)$$

where μ is the mean vector of the random field, and $\mathbf{C}$ is the covariance matrix with entries given by the covariance between points i and j equal to $C_{i,j}(\tau_{i,j})$ where $\tau_{i,j}$ is either the separation distance or a lag-vector between points i and j. If $\tau_{i,j}$ is the distance between points i and j, the field is isotropic. The covariance function $C_{i,j}(\tau_{i,j})$ can be expressed in terms of an underlying correlation function, $\rho_{i,j}(\tau_{i,j})$, and the standard deviations of the Gaussian distributions at each point, σ_i and σ_j, such that $C_{i,j}(\tau_{i,j}) = \sigma_i\sigma_j\rho_{i,j}(\tau_{i,j})$.

The covariance function need not be isotropic. $C_{i,j}$ can be found using the directional components of $\tau_{i,j}$ if this quantity is computed as a vector with magnitude equal to the distance between i and j. A random field can be considered weakly homogeneous if the mean and standard deviation are invariant across the field [34]. In this paper, only fields of this type are considered.

2.3. Local averaging of random fields for field discretisation

To simulate random fields, it is necessary to discretise the infinite set of points that the field represents mathematically into a representation suitable for use with computers. Several methods exist for the discretisation of a random field, Sudret and Der Kiureghian [40] contains an extensive review. In this paper, a local averaging, see [44], approach is adopted in order to discretise a random field onto a triangular finite element mesh. The modelling of engineering property parameters by local averages is discussed at length in Fenton and Griffiths [14]. The argument for this can be summarised by considering that most engineering property parameters used for modelling macro-scale problems are poorly defined for an infinitesimally small domain. Modulus and porosity are examples of this.

In order to discretise a random field by local averaging two formulations must be known, namely the variance reduction function and the covariance between local averages. Locally averaging over some finite element reduces the variance of a discretised random field when compared to the original infinite joint distribution. If this filtering process is not applied, the correlation structure of the random field will not be correctly modelled on the discretised field. Analogous to techniques common in digital signal processing, small scale fluctuations much smaller than the scale of the discretisation must be filtered out. This ensures that, when sampling