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Research Paper

A shallow constitutive law-based granular flow model for avalanches

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ABSTRACT

In this paper, we analyze the constitutive laws of a hydraulic dynamic model, the Savage–Hutter (S–H) model, and the Hungr model. To overcome their limitations, we adopt a frictional viscoplastic constitutive law for dense granular flow and propose a new constitutive law-based dynamic model for avalanches by simplifying the internal stresses for shallow granular flow. The range of the earth pressure coefficient in each model is studied and compared. Finally, we compare the previous experimental results with those obtained from the proposed model and the S–H model, and we find that the new model can capture the flow properties of an avalanche.

1. Introduction

Landslides, debris flows, and mudflows are types of natural motion that occur under the influence of gravity [1,2]. These phenomena are distinguished by different proportions of water and sediment content in the moving mass, and they are collectively referred to as avalanches in this paper. The flow process of an avalanche can be modeled by either discrete approaches or continuum approaches. The discrete approaches, such as the distinct element method (DEM) and discontinuous deformation analysis (DDA), focus on studying individual particles that obey the basic laws of motion [3–8] and can thus describe the global properties of granular material. However, there are computational limitations to simulating avalanches that are composed of large numbers of grains, and it is not an easy task in discrete element simulations to relate the microscopic parameters between the elements to macroscopic parameters of geomaterials.

In the continuum approach to modeling the process of an avalanche, the moving mixture of sediment and water can be treated as a continuous fluid, allowing us to introduce the Navier–Stokes (N–S) equations. Additionally, if the flow depth is relatively small compared with the lateral spreading scale of the avalanche, the lateral velocity can be treated as identical along the flow depth. Therefore, the N–S equations can be integrated from free surface to base to obtain two hyperbolic partial differential equations for the depth and velocity distributions.

At present, there are many different avalanche dynamic models, and different constitutive relationships and basal friction terms have been adopted to establish the depth-integrated equations. Dambreak and flood routing models were first used to model landslides and debris flows, e.g., Lang and Brown [9], Jeyapalan [10], Takahashi [11], O'Brien et al. [12], Voight and Sousa [13], and Shieh et al. [14]. These avalanche models are known as hydraulic models. The hydrodynamic approach of such methods assumes that the internal stresses satisfy the hydrostatic state, and thus shallow-water-type equations are adopted. However, this assumption is only valid for avalanches with high water content, i.e., avalanches in the dilute state. Savage and Hutter [15] first introduced the concept of an earth pressure coefficient to describe the relative magnitude of the vertical and lateral stresses in a dry cohesionless granular flow. Although Savage–Hutter (S–H) theory is widely used, its constitutive relationship cannot reflect the flow properties of an avalanche, e.g., the velocity and strain rate, and these variables are known to be significant factors in the constitutive relationship of granular flows [16]. Hungr [17] and McDougall and Hungr [18] proposed a strain-related expression for the earth pressure coefficient based on conventional loading and unloading experiments with soil. In this paper, we examine and discuss the constitutive laws of different avalanche dynamic models (i.e., the hydraulic model, S–H model, and Hungr model). We then adopt a frictional viscoplastic constitutive law for dense granular flow [19] and propose an avalanche flow model based on shallow granular flow. Using the results from an experimental example, the S–H model is compared with that proposed in this paper.

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2. Constitutive relationships in different avalanche dynamic models

The constitutive relationship, which is a vital component in describing the moving process of an avalanche, remains an open problem. For the convenience of analysis, this paper defines the earth pressure coefficient of different avalanche dynamic models as the ratio between the lateral stress and the vertical stress. In hydraulic dynamic models, the lateral stress is assumed to be equal to the vertical stress, i.e., the earth pressure coefficient is 1. However, this is an oversimplification, as the hydrostatic assumption is not valid in avalanches with low water content. In an attempt to describe the constitutive relationship in an avalanche, the S–H and Hungr models apply a different theoretical basis to produce two other categories of earth pressure coefficient. In the next subsection, we summarize and analyze the earth pressure coefficients in these two models.

2.1. S–H model

Savage and Hutter [15] and Hutter et al. [20] developed the widely used S–H theory under the following assumptions: (1) the granular flow obeys the Mohr–Coulomb criterion; (2) the granular flow is incompressible; (3) the depth of the avalanche is shallow; (4) the basal friction is Coulomb-type friction; (5) the stresses in the downslope direction are dominant.

The theory is composed of a mass conservation equation and two momentum conservation equations [20]:

$$\frac{\partial h}{\partial t}+\frac{\partial (hu)}{\partial x}+\frac{\partial (hv)}{\partial y}=0 \tag{1a}$$

$$\frac{\partial}{\partial t}(hu)+\frac{\partial}{\partial x}\left(hu^2\right)+\frac{\partial}{\partial y}(huv)=gh\sin\theta-sgn(u)\mu' gh\cos\theta-gh\cos\theta\frac{\partial b}{\partial x}-K_x gh\cos\theta\frac{\partial h}{\partial x} \tag{1b}$$

$$\frac{\partial}{\partial t}(hv)+\frac{\partial}{\partial y}\left(hv^2\right)+\frac{\partial}{\partial x}(huv)=-sgn(v)\mu' gh\cos\theta-gh\cos\theta\frac{\partial b}{\partial y}-K_y gh\cos\theta\frac{\partial h}{\partial y} \tag{1c}$$

where h is the flow depth, u and v are the components of flow velocity in perpendicular directions, μ' is the friction coefficient between the avalanche and the base, and θ is the angle of inclination. $K_{x/y}$ is the earth pressure coefficient, expressed in the x- and y-directions as:

$$K_{x_{act/pas}}=2\sec^2\phi\left(1\mp\left(1-\cos^2\phi/\cos^2\delta\right)^{1/2}\right)-1 \tag{2a}$$

$$K_{y_{act/pas}}=\frac{1}{2}\left\{K_x+1\mp\left[(K_x-1)^2+4\tan^2\delta\right]^{1/2}\right\} \tag{2b}$$

in which ϕ is the internal angle of the granular material. δ is the friction angle between the avalanche and the base.

In S–H theory, the assumption of lateral confinement pressure is only valid when the "downhill" velocity and its variation are much larger than those in the lateral direction. S–H theory admits a simple constitutive law whereby the lateral stress depends only on the normal stress and the earth pressure coefficient. However, granular experiments by da Cruz et al. [16] have revealed that the velocity and strain rate affect the internal stress of granular flow.

To eliminate the limitations in the S–H model, several improvements have been made by various researchers. For example, Gray et al. [21] modified S–H theory to model landslides flowing over irregular three-dimensional terrain, and Bouchut et al. [22] proposed a model with relaxed restrictions on variations in slope. Bouchut et al. [23] made a further improvement by taking the erosion process into account, and Fernández-Nieto et al. [24] developed a new S–H-type theory to simulate submarine avalanches and the resultant tsunamis.

2.2. Hungr model

In contrast to the S–H model, Hungr [17,18] proposed a strain-related expression for the earth pressure coefficient, i.e., $K_j=K'_j+S_c\Delta\varepsilon_j$. In this expression, the stiffness coefficient S_c is taken to be $S_c=(K_p-K_a)/0.05$ under the loading condition and $S_c=(K_p-K_a)/0.025$ under the unloading condition. K_p and K_a are the active and passive values of the earth pressure coefficient, respectively. $\Delta\varepsilon_j$ is the incremental tangential strain in each divided column and is defined as

$$\Delta\varepsilon_j=\frac{(S_{i+1}-S_i)-(S'_{i+1}-S'_i)}{S'_{i+1}-S'_i} \tag{3}$$

where S_i is the displacement of column i in the current step, and S'_i is the displacement of column i in the previous step.

In the above expression for the pressure coefficient, i.e., $K_j=K'_j+S_c\Delta\varepsilon_j$, the incremental tangential strain of a specific column is determined by its own displacement and that of the adjacent column. Thus, rewriting Eq. (3) with the flow velocity as the variable, we have

$$\Delta\varepsilon_j=\frac{\frac{S_{i+1}-S_i}{\Delta t}-\frac{S'_{i+1}-S'_i}{\Delta t}}{S'_{i+1}-S'_i}\cdot\Delta t=\frac{\frac{S_{i+1}-S'_{i+1}}{\Delta t}-\frac{S_i-S'_i}{\Delta t}}{S'_{i+1}-S'_i}\cdot\Delta t=\frac{u_{i+1}-u_i}{\Delta x_i}\cdot\Delta t =\frac{\partial u_i}{\partial x}\cdot\Delta t \tag{4}$$

The earth pressure coefficient can then be expressed as

$$K_j=K'_j+S_c\Delta\varepsilon_j=K'_j+S_c\cdot\Delta t\frac{\partial u_i}{\partial x}=K'_j+a_{c/u}\frac{\partial u_i}{\partial x} \tag{5}$$

In Eq. (5), the parameter $a_{c/u}$ is defined as $a_c=\frac{\Delta t}{0.05}(K_p-K_a)$ under compression and $a_u=\frac{\Delta t}{0.025}(K_p-K_a)$ under unloading. Additionally, it can be observed from Eq. (5) that the earth pressure coefficient of a particular column is determined by the earth pressure coefficient in the previous time step, the velocity gradient of the column, the size of the time step, and the difference between the critical earth pressure coefficients.

3. Model development

3.1. Constitutive relationship

Jop et al. [19] proposed a constitutive relationship for granular flow in which the internal stresses of the granular flow are related to the isotropic pressure, the strain rate, and the second invariant of the strain rate. In addition, they proposed a definition of effective viscosity to describe the flow properties, and they considered the theoretical effect of velocity on the magnitude of this effective viscosity. Here, we put forward a simplified form of the constitutive relationship given by Jop et al. [19]. This form is suitable for shallow flows, and it enables us to build a constitutive law-based model for shallow granular flows.

Consider a granular material flowing on an inclined plane. We analyze a cubic element in the flow with dimensions of $dx\times dy\times h$ (Fig. 1). During the flow process, if the scale of lateral spreading is much larger than that of the flow depth and the lateral velocity is much greater than the vertical velocity, then the granular flow can be treated as a "shallow" flow. In shallow granular flows, the gradient of vertical velocity is small, and the shear stresses on the x–z and y–z planes can thus be neglected. The remaining stresses are shown in Fig. 1, where τ_{xz} and τ_{yz} are shear stresses generated by basal friction and p_{xx}, p_{yy}, and p_{zz} are the normal stresses along the three coordinate axes. According to the constitutive relationship proposed by Jop et al. [19], the three normal