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Research Paper

On spatial averaging along random slip lines in the reliability computations of shallow strip foundations

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A B S T R A C T

This paper presents a new approach to the reliability analysis of shallow foundations in which soil strength parameters are considered as random fields. These fields were averaged along the slip lines that resulted from Prandtl's mechanism in accordance with Vanmarcke's approach. An efficient algorithm for evaluating reliability measures when the bearing capacity along kinematically admissible slip lines has been investigated. A significant improvement relative to earlier works is obtained by utilising random slip line locations. This improvement allows authors to evaluate the impact on reliability indices of incorporating changes in slip line geometry. Finally, reliability indices were evaluated for foundations of various widths by Monte Carlo simulation, and the results were compared with those obtained in earlier works.

1. Introduction

The spatial variability of soil properties distinguishes geotechnical engineering from other areas of civil engineering. Physical and mechanical parameters vary randomly within even homogeneous layers of soil deposits. These variations are consequences of the diversity of geological processes that occurred in the past as well as the various sedimentation and consolidation processes of the upper layers [1]. Consequently, a probabilistic approach seems quite appropriate when investigating foundation safety. Conversely, it is well-known that each mechanism associated with foundation failure is affected by a certain subsoil domain with a specific volume. From a probabilistic view point, this fact means that a given soil property could be adequately described by a random field. Important progress in the application of mathematical random fields theory was achieved by Vanmarcke in his fundamental works [2–4]. He proposed that an initial random field should be averaged over an area in a manner dependent on the geotechnical context. In many later papers, authors have suggested that credible reliability assessment for structures interacting with soil requires the spatial averaging of soil properties (e.g., [5–7]).

Earlier experiences described by the first author, Puła [8,9], revealed that when no spatial averaging procedure was applied, reliability computations conducted in the context of the foundation's bearing capacity yielded significantly low reliability index values (large failure probability values) even for foundations considered relatively safe. However, the volume of the area subjected to averaging strongly affects the results of reliability computations. Hence, the selection of the averaged area is a vital problem and must consider the failure mechanism. One possibility (proposed in papers [10,8,9]) is to study the spatial averages associated with the kinematically admissible mechanism of failure proposed by Prandtl [11]. Soil strength parameters are assumed to constitute anisotropic random fields with different vertical and horizontal fluctuation scales. These fields are subjected to averaging along potential slip lines within the given mechanism. However, in earlier papers [8,9], the potential slip lines in the Prandtl mechanism were not subjected to random variations and were kept constant during the computations. The constant location of a slip line is based on the expected value of the internal friction angle (due to the stationarity assumption, the expected value was constant). Meanwhile, the use of a fixed slip line position seems to be controversial. In a discussion in the CISM course in Udine, Italy, Vaughan D. Griffiths has suggested that the use of non-random slip lines may induce significant error in reliability measure values because the line geometry depends on the angle of internal friction of the subsoil [private communication with Vaughan D. Griffiths]. This discussion inspired the investigation of the influence of the random character of slip lines on the reliability assessments presented in this paper. The important role of the random character of slip

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lines was also reported in the paper of Popescu et al. [12]. The random selection of slip lines implies conducting an averaging procedure for each individual slip line. This selection can be performed by a simulation procedure dedicated to this task. The current paper presents a new numerical algorithm for the evaluation of the probability of failure under the Prandtl failure mechanism. The algorithm takes into account the random character of the slip line positions caused by random variations of the angle of internal friction in the subsoil. A series of reliability computations based on this new algorithm were carried out. The results are compared with those obtained in a previous study [9].

2. Bearing capacity evaluation based on Prandtl's mechanism

This paper addresses Prandtl's failure mechanism. The bearing capacity of shallow foundations, which was introduced by Prandtl, was one of the first issues solved using bearing capacity theory [11]. The formula that describes the limiting value of the bearing capacity originates from the theorem concerning the evaluation of an upper boundary load. The theorem states that the work performed by the loads cannot exceed the energy dissipated by the failure mechanism. A schematic representation of Prandtl's mechanism is presented in Fig. 1.

In this paper, using the spatial averaging procedure proposed by Vanmarcke [2,4], soil strength parameters are subject to averaging on slip lines AB, BC and CD individually (Fig. 1). Using parameters $\varphi_1, c_1, \varphi_2, c_2, \varphi_3$ and c_3 along AB, BC and CD and comparing the work of the external load with the total dissipation of internal energy, the value of the limit load (bearing capacity) can be established in the same manner as in the classical approach. The approach given below is based on the evaluation presented by Izbicki and Mróz [13] and was adopted for the case of three independent lines from a study by Puła [8]. The bearing capacity is the sum of three components: the effect of the weightless cohesive soil (Q_1), the effect of loading in the vicinity of foundation (Q_2), and the effect of the self-weight of the soil (Q_3) The final equations are as follows:

$$Q_f = \frac{bQ_0}{\sin\left(\frac{\pi}{4} - \frac{\varphi_1}{2}\right)} \tag{1}$$

where

$$Q_0 = Q_1 + Q_2 + \frac{b}{2}(Q_{31} + Q_{32} + Q_{33}) \tag{2}$$

and

$$Q_1 = c_1 \frac{\cos\varphi_1}{2\sin\left(\frac{\pi}{4} - \frac{\varphi_1}{2}\right)} + c_2[\exp(\pi\tan\varphi_2) - 1]\frac{1}{2\sin\left(\frac{\pi}{4} - \frac{\varphi_1}{2}\right)\tan\varphi_2} + c_3\exp(\pi\tan\varphi_2)\frac{\cos\varphi_3}{2\sin\left(\frac{\pi}{4} - \frac{\varphi_1}{2}\right)} \tag{3}$$

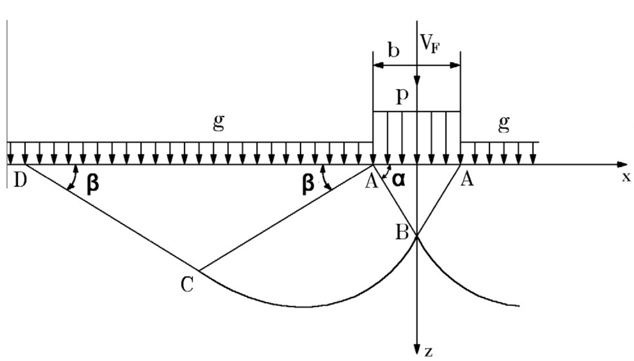

Fig. 1. Schematic presentation of Prandtl's mechanism (reflection in the z axis), where *angles* : $\alpha = \frac{\pi}{4} + \frac{\varphi}{2}; \beta = \frac{\pi}{4} - \frac{\varphi}{2}$.

$$Q_2 = (g + \gamma D)\exp(\pi\tan\varphi_2)\cos\left(\frac{\pi}{4} - \frac{\varphi_1}{2}\right)\frac{\cos\left(\frac{\pi}{4} + \frac{\varphi_1}{2} - \varphi_3\right)}{\sin\left(\frac{\pi}{4} - \frac{\varphi_1}{2}\right)} \tag{4}$$

$$Q_{31} = -\frac{1}{4}\gamma\cos\left(\frac{\pi}{4} - \frac{\varphi_1}{2}\right) \tag{5}$$

$$Q_{32} = \frac{\gamma}{2(1 + 9\tan^2\varphi_2)4\sin^2\left(\frac{\pi}{4} - \frac{\varphi_1}{2}\right)} \times \left\{\left[3\tan\varphi_2\sin\left(\frac{\pi}{4} + \frac{\varphi_1}{2}\right) - \cos\left(\frac{\pi}{4} + \frac{\varphi_1}{2}\right)\right] \times \exp\left(\frac{3}{2}\pi\tan\varphi_2\right) + \left[3\tan\varphi_2\sin\left(\frac{\pi}{4} - \frac{\varphi_1}{2}\right) + \cos\left(\frac{\pi}{4} - \frac{\varphi_1}{2}\right)\right]\right\} \tag{6}$$

$$Q_{33} = \frac{\gamma\cos\varphi_3\exp\left(\pi\frac{3}{2}\tan\varphi_2\right)\cos\left(\frac{\pi}{4} + \frac{\varphi_1}{2} - \varphi_3\right)}{8\sin^2\left(\frac{\pi}{4} - \frac{\varphi_1}{2}\right)} \tag{7}$$

where γ is the unit weight of soil, D is the depth of the foundation, g is the overburden pressure and b is the width of the foundation. The multiplier of 0.5 in Eq. (2) was used to obtain a limit load (bearing capacity) value similar to the solution proposed by Sokołowski [14], which is more suitable for use in engineering computations.

3. Characterisation of soil properties by random fields oriented for reliability computations

3.1. The concept of spatial averaging

Assume that a soil property X, which is considered to be random, can be described by a stationary random field with a covariance function $R(\Delta x, \Delta y, \Delta z) = \sigma_X^2 \rho(\Delta x, \Delta y, \Delta z)$, where σ_X^2 denotes the variance (point variance) of X and ρ is the correlation function. Let $V \subset \boldsymbol{R}^3$ be a domain and $|V|$ denote its measure (volume). Spatial averaging, which was proposed by Vanmarcke [2], introduces a new random field (moving average random field) defined by the following equation:

$$X_V = \frac{1}{|V|}\iiint_V X(x,y,z)dx\,dy\,dz \tag{8}$$

Due to the stationary property, the new random field X_V preserves the same mean value; however, its variance and covariance function are subjected to changes depending on the choice of V. The variance of the new random field X_V is given by

$$VAR[X_V] = \sigma_V^2 = \gamma(V)\sigma_X^2 \tag{9}$$

where $\gamma(V)$ is a variance function or variance reduction function describing changes in the point variance σ_X^2 after applying the spatial average procedure.

3.2. Reliability

Reliability theory concerns the basic vector of random variables $\boldsymbol{Z} = (Z_1, \ldots, Z_n)$, the coordinates of which are random variables that characterise the scenario, e.g., ground parameters, construction features and loads. This vector is an argument of the limit state function, which is defined as

$$g(Z) = \left\{\begin{array}{l} \geqslant 0 \text{ for the safe state of the structure,} \\ < 0 \text{ for the failure state of the structure} \end{array}\right\} \tag{10}$$

In this paper, $g(Z)$ is as follows:

$$g(Z) = Q_f - F, \tag{11}$$

where Q_f is the bearing capacity defined in Eq. (1) and F is the sum of the actions (loads) that contribute to the loss of equilibrium. As a reliability measure, the probability of failure is used: