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Research Paper

Bivariate distribution of shear strength parameters using copulas and its impact on geotechnical system reliability

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ABSTRACT

The objective of this paper is to investigate the effect of copulas for constructing the bivariate distribution of shear strength parameters on system reliability of geotechnical structures. First, the bivariate distribution of shear strength parameters is constructed using copulas. Second, the implementation procedure of system reliability analysis using direct Monte Carlo simulation (MCS) is developed. Finally, the system reliability of a retaining wall and a rock wedge slope is presented to explore the effect of copula selection on geotechnical system reliability. The results indicate that the system reliability of geotechnical structures under incomplete probability information could not be determined uniquely because the bivariate distribution of cohesion and friction angle with given marginal distributions and correlation coefficient could not be determined uniquely. The copulas for modeling dependence structure between cohesion and friction angle have a significant influence on the system reliability of geotechnical structures. Such an influence includes two phases separately. The first phase is that the dependence structure between shear strength parameters characterized by copulas affects the reliability of single failure mode, depending on the marginal distributions, dependence structure between shear strength parameters, and reliability level of each failure mode. The second phase is that the reliability of each failure mode influences on system reliability, only depending on reliability level of each failure mode and correlations among various failure modes. It is important to distinguish between the effect of copula selection on reliability of each failure mode and that on geotechnical system reliability.

1. Introduction

The shear strength parameters [cohesion (c') and friction angle (ϕ')] are important parameters for reliability analysis of geotechnical structures, such as slopes, retaining walls, and strip footings [1,2,6,12–14,23,26,27,32]. To achieve a realistic evaluation of geotechnical reliability, the joint cumulative distribution function (CDF) or probability density function (PDF) of the shear strength parameters should be known. In geotechnical engineering practice, however, the joint CDF or PDF is often unknown due to limited data from field test or laboratory test. On the basis of these limited data, only the marginal distributions and correlation coefficient underlying the shear strength parameters can be determined, which are referred to as incomplete probability information in this study. It is concluded that the joint probability distribution of shear strength parameters under incomplete probability information could not be determined uniquely [10,46,52].

Recently, copula theory (e.g., [43]) has been applied to construct the joint probability distribution of correlated geotechnical parameters under incomplete probability information. For example, Li et al. [33,34] used the copula approach to construct the joint PDF of two curve-fitting parameters underlying load–displacement curves of piles. Uzielli and Mayne [47,48] investigated the dependence between load–displacement model parameters underlying vertically loaded shallow footings on sands using copula. Tang et al. [46] investigated the impact of copula selection on slope and retaining wall reliability. Wu [49] employed the Gaussian and Frank copulas to model the trivariate distribution among cohesion, friction angle and unit weight of soils. Huffman and Stuedlein [19] adopted several copulas for modeling the measured dependence structure between the coefficients of the two-parameter

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Table 1
Summary of the adopted bivariate copula functions in this study.

Copula	Copula function, $C(u_1, u_2; \theta)$	Copula density function, $D(u_1, u_2; \theta)$	Generator function, $\phi_\theta(t)$	Range of θ
Gaussian	$\Phi_\theta(\Phi^{-1}(u_1), \Phi^{-1}(u_2))$	$\frac{1}{\sqrt{1-\theta^2}}\exp\left[-\frac{\varsigma_1^2\theta^2-2\theta\varsigma_1\varsigma_2+\varsigma_2^2\theta^2}{2(1-\theta^2)}\right]$, $\varsigma_1=\Phi^{-1}(u_1)$ $\varsigma_2=\Phi^{-1}(u_2)$	–	[−1, 1]
Plackett	$\frac{S-\sqrt{S^2-4u_1u_2\theta(\theta-1)}}{2(\theta-1)}$, $S=1+(\theta-1)(u_1+u_2)$	$\frac{\theta[1+(\theta-1)(u_1+u_2-2u_1u_2)]}{\{[1+(\theta-1)(u_1+u_2)]^2-4u_1u_2\theta(\theta-1)\}^{\frac{3}{2}}}$	–	$(0, \infty)\backslash\{1\}$
Frank	$-\frac{1}{\theta}\ln\left[1+\frac{(e^{-\theta u_1}-1)(e^{-\theta u_2}-1)}{e^{-\theta}-1}\right]$	$\frac{-\theta(e^{-\theta}-1)e^{-\theta(u_1+u_2)}}{[(e^{-\theta}-1)+(e^{-\theta u_1}-1)(e^{-\theta u_2}-1)]^2}$	$-\ln\frac{e^{-\theta t}-1}{e^{-\theta}-1}$	$(-\infty, \infty)\backslash\{0\}$
No.16	$\frac{1}{2}(S+\sqrt{S^2+4\theta})$, $S=u_1+u_2-1-\theta\left(\frac{1}{u_1}+\frac{1}{u_2}-1\right)$	$\frac{1}{2}\left(1+\frac{\theta}{u_1^2}\right)\left(1+\frac{\theta}{u_2^2}\right)S^{-\frac{1}{2}}\left\{-S^{-1}\left[u_1+u_2-1-\theta\left(\frac{1}{u_1}+\frac{1}{u_2}-1\right)\right]^2+1\right\}$, $S=\left[u_1+u_2-1-\theta\left(\frac{1}{u_1}+\frac{1}{u_2}-1\right)\right]^2+4\theta$	$\left(\frac{\theta}{t}+1\right)(1-t)$	$[0, \infty)$

Note: The symbol "–" denotes that the generator function is not available; Φ^{-1} is the inverse standard normal distribution function; Φ_θ is the bivariate standard normal distribution function with Pearson linear correlation coefficient θ.

bearing pressure–displacement model. Tang et al. [44] proposed three copula-based approaches to evaluate slope reliability under incomplete probability information. These studies indicate that the copulas provide a fairly general method for constructing multivariate distributions that satisfy some non-parametric measure of dependence and the prescribed marginal distributions.

Constructing the multivariate distributions of correlated geotechnical parameters using copulas is an important step for evaluating reliability of geotechnical structures. The impact of the multivariate distributions of correlated geotechnical parameters using various copulas on geotechnical reliability may be of interest. For this reason, Li et al. [33] explored the influence of copula selection on serviceability limit state reliability of piles. Tang et al. [46] investigated the effect of copula selection on reliability of an infinite slope and a retaining wall. These studies concluded that the copula selection has a significant influence on the probabilities of failure of geotechnical structures. Note that the aforementioned studies only focused on the reliability of single failure mode underlying the geotechnical structures. However, geotechnical structural systems usually consist of more than one failure mode (e.g., [7,18,20–22,35,40,51]). It is evident that the reliability of single failure mode underlying geotechnical structures could not represent the system reliability of the geotechnical structures. It is of practical interest to distinguish between the reliability of each failure mode and the system reliability of the entire geotechnical structural system. With these in mind, the effect of copulas for modeling the joint distribution of shear strength parameters on the system reliability of geotechnical structures should be explored, and is the topic of the present research.

This paper aims to explore the effect of copulas for constructing the bivariate distribution of shear strength parameters on system reliability of geotechnical structures under incomplete probability information. First, the bivariate distribution of shear strength parameters is constructed in the copula framework. Four copulas, namely Gaussian, Plackett, Frank, and No.16 copulas (e.g., [43]), are selected to model the dependence structure between cohesion and friction angle. Second, the implementation procedure of system reliability calculation using direct Monte Carlo simulation (MCS) is developed. Finally, the system reliability of a retaining wall and a rock wedge slope is presented to demonstrate the effect of copula selection on geotechnical system reliability.

2. Bivariate distribution of shear strength parameters using copulas

As mentioned in the introduction, this paper will adopt copulas for modeling the bivariate distribution of shear strength parameters. Copulas are functions that couple a multivariate distribution function to its one-dimensional marginal distribution functions. Alternatively, copulas are multivariate distribution functions whose one-dimensional marginal distribution functions are uniform on the interval of [0,1] (e.g., [43]). There are many copulas in the literature such as Gaussian, t, Plackett, Frank, Gumbel and Clayton copulas. Each copula is characterized by its own dependence structure. According to Sklar's theorem (e.g., [43]), the bivariate distribution, $F(c', \phi')$, of the two shear strength parameters c' and ϕ' can be expressed in terms of a copula function $C(u_1, u_2; \theta)$ and the marginal distributions $u_1 = F_1(c')$ and $u_2 = F_2(\phi')$:

$$F(c', \phi') = C(F_1(c'), F_2(\phi'); \theta) = C(u_1, u_2; \theta) \tag{1}$$

where θ is a copula parameter describing the dependency between c' and ϕ'. From Eq. (1), the bivariate PDF, $f(c', \phi')$, of c' and ϕ' can be obtained as (e.g., [43])

$$f(c', \phi') = f_1(c')f_2(\phi')D(F_1(c'), F_2(\phi'); \theta) \tag{2}$$

where $D(F_1(c'), F_2(\phi'); \theta)$ is a copula density function, which is given by

$$D(F_1(c'), F_2(\phi'); \theta) = D(u_1, u_2; \theta) = \partial^2 C(u_1, u_2; \theta)/\partial u_1 \partial u_2 \tag{3}$$

It is evident that both the copula function $C(u_1, u_2; \theta)$ and the copula density function $D(u_1, u_2; \theta)$ are related to the copula parameter θ. The copula parameter θ can be determined through the Pearson correlation coefficient ρ between c' and ϕ'. According to the definition of Pearson correlation coefficient (e.g., [4]), the integral relationship between ρ and θ can be expressed as follows:

$$\rho = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\frac{c'-\mu_{c'}}{\sigma_{c'}}\right)\left(\frac{\phi'-\mu_{\phi'}}{\sigma_{\phi'}}\right)f_1(c')f_2(\phi')D(F_1(c'), F_2(\phi'); \theta)\mathrm{d}c'\mathrm{d}\phi' \tag{4}$$

where $\mu_{c'}$ and $\mu_{\phi'}$ are the means of c' and ϕ', respectively; $\sigma_{c'}$ and $\sigma_{\phi'}$ are the standard deviations of c' and ϕ', respectively. For prescribed marginal distributions of c' and ϕ', and correlation coefficient ρ between c' and ϕ', the preceding integral equation can be solved iteratively to obtain θ. For instance, a two-dimensional Gaussian–Hermite integral technique, developed by Li et al. [36], can be used for such a purpose.

When the probability information on shear strength parameters is only limited to marginal distributions and a correlation coefficient, a large number of copulas that are consistent with such information can be used to characterize the dependence structure. Since there exists a negative correlation between c' and ϕ' (e.g., [31,37,45,46]), the copulas that allow a wide range of negative correlation coefficients are selected to characterize the dependence between c' and ϕ'. A review of the literature reveals that the Gaussian copula, Plackett copula, Frank copula and No.16 copula (e.g., [43]) are appropriate for describing the dependence structure