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A dual-scale approach to model time-dependent deformation, creep and fracturing of brittle rocks

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ABSTRACT

A physically-motivated dual-scale modeling approach is proposed to model the time-dependent damage, deformation and fracturing behavior of heterogeneous brittle rocks during creep. The proposed model uses a microcrack-based damage constitutive law established at the elemental scale, in which the time-dependent degradation of elastic stiffness and damage-induced anisotropy are directly linked to microcrack growth. The evolution of mechanical heterogeneity is based on a Weibull distribution that captures the transition from distributed damage to localized failure. The key feature of the proposed model is to establish an adequate prediction of macroscopic creep behavior based on the microscopic kinetics of microcrack growth rather than the phenomenological material degradation laws adopted in previously-developed statistical models. The general capabilities of the proposed model are illustrated with numerical simulations of biaxial creep tests. The influences of differential stresses, heterogeneities and microscopic element sizes on creep behavior in brittle rocks are also examined. Results from such analyses indicate that the proposed model not only accurately replicates the trimodal phases of creep deformation and the associated temporal evolution of acoustic emission but also follows the progressive evolution of fracture modes and morphology commonly observed. Thus, subject to suitable calibration, this model provides an attractive virtual experimental tool to probe process-based understanding of complex long-term problems related to structures on an in geologic media.

1. Introduction

Understanding creep response of rocks in the Earth's crust is but one of many challenges that relate to characterizing natural and engineered processes related to earthquake rupture, the triggering of volcanic eruptions, failure in underground mining, hydrocarbon and geothermal energy recovery and hazardous waste disposal, among others [1]. The modeling of time-dependent deformation and fracturing of rocks under creep conditions is fundamental to better assessing the precursory phenomena of geohazards or predicting the long-term stability and safety of engineering facilities.

In the past several decades, extensive laboratory creep experiments have been performed to investigate the time-dependent behaviors of many kinds of rocks (*e.g.* [2–6]). Typical experimental observations indicate that rock specimens deforming under a constant stress over extended periods of time generally exhibit a trimodal creep behavior, *i.e.* primary or transient creep, followed by secondary or steady-rate creep, and terminating in tertiary or accelerating creep that eventually progresses to dynamic rupture [7–10]. To represent such time-dependent behavior of rocks, a variety of theoretical and numerical models have been proposed. These models can be generally divided into two categories: (i) phenomenological approaches and (ii) micromechanics-based approaches [11,12]. Phenomenological models are developed on the basis of empirically observed internal variables (*e.g.* [13]) or the superposition of several viscous and elastic elements (*e.g.*, the Burgers body model [14]). Such models can produce the macroscopically observed creep curves of rocks by fitting with experimental data (*e.g.* [15,16]). However, the inherent physical mechanisms related to creep deformation are not accommodated in these models, so the key mechanistic parameters remain physically unclear.

It is generally accepted that the time-dependent non-linear deformation and fracturing process that evolve in brittle rocks are mainly dominated by the progressive accumulation of subcritical damage originating from the propagation and coalescence of microcracks due to stress corrosion occurring at crack tips [1,17,18]. To overcome the disadvantages encountered

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in phenomenological models, it is necessary to establish a link between the subcritical processes of microcracking and its impact on the observed macroscopic creep response [1]. Thus, micromechanics-based models, based on a suitable analysis of subcritical microcracking within fracture mechanics, may contribute this linkage between mechanism and observation. Several representative models developed from such micro-scale considerations are briefly reviewed here. Costin [19] and Shao et al. [20–22] proposed a microcrack model to describe the time-dependent deformation and failure of brittle rock by introducing subcritical damage laws. Yoshida and Horii [23] derived an analytical model of stress corrosion cracking based on a sliding wing crack model [24] and implemented it into a finite element program to predict the creep behavior of underground structures. Golshani et al. [25] proposed a micromechanics-based numerical model to examine time-dependent microcracking and the development of excavation damage zone (EDZ) around an opening. Konietzky et al. [26] developed a new numerical cellular automata approach based on subcritical crack growth to predict the life-time of rocks and Brantut et al. [27] extended the classical micromechanical model of Ashby and Sammis [28] by accommodating subcritical cracking to describe the time-dependent creep behavior of water-saturated rocks. These micromechanics-based models make a significant contribution in establishing the connection between microcrack kinetics and the resulting macroscopically time-dependent responses, but most of them only provide the temporal evolution of deformation and damage during creep, and cannot capture the spatial distribution and localization of deformation and damage. Such information is of particular importance in understanding the fracturing processes of rocks [1,9].

To model both the temporal and spatial evolution of deformation and damage in rocks during creep, a few notable statistical models have been constructed. Amitrano and Helmstetter [9] developed a numerical approach based on FEM to model the time evolution of strain (three creep phases), as well as the progressive damage localization before failure, by introducing an empirical time-to-failure law and the intrinsic heterogeneities of rocks at the micro-scale. Xu et al. [29,30] also developed a similar model by extending the RFPA model of Tang et al. [31,32], in which the exponential degradation laws of material strength and Young's modulus were used to represent the time-dependent rheological behavior of basic elements of heterogeneous brittle rocks. Such statistical models have been successful in describing many of the features of creep phenomenology both in time and in space. However, the major difficulty with these models arises from the use of *ad hoc* assumptions on the local time-dependent constitutive laws in their formulation. These *defacto* surrender insightful descriptions of microscopic kinetics of microcrack growth with the net effect that it is difficult to accurately evaluate the true mechanisms of rock degradation as a function of time-dependent damage evolution. Accordingly, by nature, these statistical models are phenomenological [1], but nevertheless they provide a new way for modeling the creep behavior of rocks.

In this paper, we develop a new physically-motivated dual-scale model for the time-dependent deformation and fracturing behavior of brittle rocks. This model is an extension of our previous model [33] in which only stress-induced (time-independent) microcracking was incorporated to analyze the fracturing evolution in brittle rocks. By accommodating the physical process of subcritical microcrack growth, the present model allows time-dependent deformation, damage accumulation and localization, and associated acoustic emission to be followed in brittle rocks during creep in a realistic way. The essential difference between this model and the previous statistical models [9,30] is that in the present model the time-dependent deformation and failure of brittle rocks are described in terms of the evolution of microcracks rather than *ad hoc* local phenomenological material degradation laws. In the following, a detailed description of the dual-scale modeling approach is given, and simulations of biaxial creep tests are then presented to demonstrate the capabilities of the proposed model.

2. Dual-scale modeling approach

2.1. Conceptual model

Microscopic experimental observations have shown that brittle rocks generally contain various grain-scale heterogeneities, such as grain boundaries, micropores and microcracks [34–36] that may act as nucleation sites. In particular, the presence of such micro-defects strongly influences the macroscopic mechanical behavior of rocks by serving as stress concentrators and leading to micro-cracking. However, it is extremely difficult to describe these complicated textures realistically. To aid a faithful mechanistic representation of the distribution of micro-defects, it is assumed here that the macroscopic rock medium can be divided into a series of regularly arranged, uniform prismatic microscopic elements, *i.e.* representative elemental volumes (REVs), as shown in Fig. 1. This hypothesis is also widely adopted in previous statistical models (*e.g.* [9,30]), but in many models the microscopic elements are commonly assumed to be homogeneous and isotropic and to follow the phenomenological constitutive laws which obscure the important features of microcracking. For this reason, in this paper it is assumed that each of the microscopic elements is not homogeneous but rather is a heterogeneous medium consisting of a rock matrix containing an arbitrary distribution of microcracks randomly oriented in space (see Fig. 1). The matrix of these microscopic elements is assumed to be an initially equivalent elastic and isotropic homogenized medium, but its initial properties vary randomly from one microscopic element to another within the macroscopic rock medium – this in turn results in the observed heterogeneity of the rock. It is also assumed that a family of idealized microcracks (line cracks in 2-D, or penny-shaped cracks in 3-D) are inserted into the matrix of the microscopic elements and each microcrack can be specified by its initial length ($2a_0$) and orientation angle (β) (in the case of the 2-D problem). Thus, the real rock medium is then represented by a dual-scale (micro–macro-scale) conceptual model: (i) at macro-scale the rock medium is composed of the microscopic elements (REVs) and (ii) at micro-scale each of the microscopic elements comprises both homogeneous matrix and a random ensemble of microcracks.

The proposed dual-scale model allows macroscopic deformation and fracturing behavior of rock to be replicated as prescribed by processes at the microscopic level. For a given numerical specimen under prescribed boundary conditions, the linear elastic FEM can be first applied to calculate the average local stress field ($\bar{\sigma}_{ij}$) around each microscopic element according to the initially assigned material properties. Then, for a microscopic element subjected to the local stress field ($\bar{\sigma}_{ij}$), damage may evolve due to either stress-induced microcracking or time-dependent microcracking – either of which lead to a net degradation of mechanical properties and finally to the rupture of this microscopic element. The governing equations describing such damage evolution are presented in the following. As a microscopic element fails its stiffness is reduced to a vanishingly small magnitude (*e.g.* 10^{-5} Pa [31]) to allow it to ultimately represent the discontinuity generated by the nucleation of multiple similar macroscopic fractures. Actually, such a modeling approach originates from the smeared crack model [37] in conventional fracture mechanics analysis. Since the resulting fracture has the width of a microscopic element, the size of the microscopic element should be proportional to the smallest