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Research Paper

Analytical solution for the two-dimensional plane strain consolidation of an unsaturated soil stratum subjected to time-dependent loading

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ABSTRACT

This paper introduces an exact analytical solution predicting variations in excess pore-air and pore-water pressures and settlement considering the two-dimensional (2D) plane strain consolidation of an unsaturated soil stratum subjected to different time-dependent loadings. Based on the proposed solution, the distributions of excess pore pressures along vertical and horizontal directions can be determined. The general solution is first expressed in a series of eigenfunctions of homogeneous partial differential equations (PDEs) and is then substituted into the governing flow equations. Using term-by-term differentiation and the orthogonality of the sine function, the governing equations become ordinary differential equations (ODEs). Once the complex domain is obtained by applying the Laplace transformation technique, the closed-form analytical solutions describing the dissipation of excess pore-air and pore-water pressures can be obtained by taking a Laplace inverse. In this study, four external loadings, including ramping, asymptotic, sinusoid and damped sine wave, are simulated and incorporated into the proposed solutions. For the data analysis, the 2D consolidation behavior is investigated against variations in the permeability ratio (k_a/k_w). Additionally, parametric studies regarding loading functions are presented in this paper.

1. Introduction

As a common issue in geotechnical engineering, consolidation is a process that reduces the soil volume due to the dissipation of excess pore pressures. The consolidation equation for saturated soils proposed by Terzaghi [1] has been a solid framework for several contemporary research projects since the mid-1920s. However, during engineering developments, there was great attention towards critical changes in the soil state, which significantly affects the reliability of the traditional soil mechanics framework. According to Fredlund [2] and Fredlund et al. [3], earth works (e.g., excavation, compaction) and the long-term climatic conditions (e.g., arid or semi-arid climates) in a region may gradually lower the groundwater table with time. This indicates that the portion of the soil deposit near the ground surface may be subjected to environmental flux boundary conditions and eventually becomes unsaturated. Such a portion of unsaturated soil is termed the vadose zone. Most mankind structures, especially light-engineered structures, are built on unsaturated ground surface, which is well above the groundwater table, and induces a certain compression on the upper vadose zone [2]. Nevertheless, these consolidation-related problems have not been comprehensively understood for decades. There was a realization that the study of unsaturated soil mechanics was much more complicated than that of fully saturated soil mechanics. The constitutive model of unsaturated soils was likewise employed with complex stress state variables to accommodate pore-air pressure, resulting in a highly nonlinear consolidation problem that requires a cumbersome evaluation for predicting purposes.

Attention has been given to consolidation studies of unsaturated soils since the early 1960s. This was evidenced by some original research proposed by Scott [4], who estimated the consolidation of unsaturated soil with occluded air bubbles, and Barden [5,6], who analyzed the consolidation of compacted and unsaturated clays. In the late 1970s, Fredlund and Hasan [7] proposed a set of continuity equations presenting the continuous flow of air and water phases in unsaturated soil in a one-dimensional (1D) field. This theory was later expanded to the 2D plane strain consolidation models by Dakshanamurthy and Fredlund [8] using the concept of 2D heat diffusion.

The past two decades have witnessed vigorous improvement in computational tools, which have provided geotechnical researchers the capability to estimate the physical behaviors of unsaturated

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Nomenclature

A	parameter influencing the asymptotic loading
a	linear loading rate
B	parameter influencing the sinusoidal loading
b	exponential loading rate for asymptotic loading
C	parameter influencing the damped sine wave loading
c	exponential loading rate for damped sine wave loading
C_a	interactive constant associated with the air phase
C_w	interactive constant associated with the water phase
$c^a_{v_x}$	coefficient of volume change with respect to the air phase in the x-direction
$c^a_{v_z}$	coefficient of volume change with respect to the air phase in the z-direction
$c^w_{v_x}$	coefficient of volume change with respect to the water phase in the x-direction
$c^w_{v_z}$	coefficient of volume change with respect to the water phase in the z-direction
c^a_σ	consolidation parameter with respect to the air phase
c^w_σ	consolidation parameter with respect to the water phase
g	gravitational acceleration
H	depth of the soil layer
i	integer for double series for domain x
j	integer for double series for domain z
K	ratio of change in the normal stresses
k_{a_x}	coefficient of permeability for air in the x-direction
k_{a_z}	coefficient of permeability for air in the z-direction
k_{w_x}	coefficient of permeability for water in the x-direction
k_{w_z}	coefficient of permeability for water in the z-direction
L	length of the soil layer
M	molecular mass of the air phase
m^a_1	coefficient of air volume change with respect to the change in net stress
m^a_2	coefficient of air volume change with respect to the change in suction
m^s_1	coefficient of volume change of soil with respect to the change in net stress
m^s_2	coefficient of volume change of soil with respect to the change in suction
m^w_1	coefficient of volume change of water with respect to the change in net stress
m^w_2	coefficient of volume change of water with respect to the change in suction
m_v	coefficient of volume change in saturated soils
n	porosity before loading
q_0	external loading
R	universal air constant
S^*	normalized settlement
$S_{\max}$	maximum ground surface settlement
S_r	degree of saturation before loading
$S(t)$	time-dependent settlement
T	time factor
$T_a(t)$	generalized Fourier coefficients varying with time with respect to pore-air pressure
$T_w(t)$	generalized Fourier coefficients varying with time with respect to pore-water pressure
t	elapsed time
u_a	pore-air pressure
$u_{a,t}$	first order of the partial differential equation (PDE) of air with respect to time
$u_{a,zz}$	second order of the partial differential equation (PDE) of air with respect to depth
u_{atm}	atmospheric pressure
u^0_a	maximum initial pore-air pressure
u_w	pore-water pressure
$u_{w,t}$	first order of the partial differential equation (PDE) of water with respect to time
$u_{w,zz}$	second order of the partial differential equation (PDE) of water with respect to depth
u^0_w	maximum initial pore-water pressure
V_0	initial volume of the soil element
V_a	volume of air within the soil element
V_w	volume of water within the soil element
x	investigated length
z	investigated depth
γ_w	water unit weight
ε_v	total volumetric strain
Θ	absolute temperature in Kelvin
θ	angular frequency for sinusoidal loading
θ°	temperature in degrees Celsius
ϑ	angular frequency for damped sine wave loading
Λ^{ij}	eigenvalue under isotropic soil
λ^{ij}_a	eigenvalue for the air phase under anisotropic soil
λ^{ij}_w	eigenvalue for the water phase under anisotropic soil
σ_x	total stress in the x-direction
σ_z	total stress in the z-direction

soils, particularly consolidation. Among these consolidation studies, there have been noticeable numerical modelling methods for both plane strain and axisymmetric problems conducted by Lloret and Alonso [9], Wong et al. [10], Conte [11], Qin et al. [12], Zhou and Tu [13], and Zhou and Zhao [14]. In addition, analytical methods have also been comparatively progressive. Most recent analytical developments were based on the concept of continuity equations given by Fredlund and Hasan [7]. By assuming constant consolidation parameters during the loading process, Qin et al. [15,16] used the Laplace transformation and Cayley–Hamilton techniques to solve the partial differential equations (PDEs) of air and water flows. The entire mathematical procedure was conducted under a constant surcharge [15] and an exponentially time-dependent surcharge [16]. On the other hand, Shan et al. [17] and Zhou et al. [18] introduced alternative terms, ψ_i and ϕ_i, respectively, which consist of excess pore-air and pore-water pressures, to convert the nonlinear inhomogeneous PDEs into traditional homogeneous PDEs and finally obtain solutions. In these studies, the unsaturated soil stratum is subjected to exponential loading [17,18] and sine wave loading [17]. The mentioned analytical methods help to validate the numerical results for the 1D consolidation of unsaturated soils under a variety of time-dependent loadings. These solutions, however, are numerically overestimated when the consolidation concept is expanded to a 2D problem, in which both horizontal and vertical drainage are considered. According to numerous research studies on the 2D consolidation for saturated soil mechanics [19,20], the dissipation of excess pore pressures during consolidation may be predominantly influenced by the horizontal drainage.

Having realized a limited analytical study about 2D plane strain consolidation system, particularly for unsaturated soils subjected to different types of loading, this paper introduces a closed-form analytical solution predicting the dissipation of excess pore-air and pore-water pressures and settlement using the continuity equations proposed by Dakshanamurthy and Fredlund [8]. The mathematical development adopts eigenfunction expansion and Laplace transformation methods along with homogeneous drainage boundary conditions and uniform initial conditions. Four different time-dependent loadings, namely, ramping, asymptotic, sinusoid and damped sine wave, are simulated and incorporated