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Research Paper

A comparison of finite volume formulations and coupling strategies for two-phase flow in deforming porous media

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ABSTRACT

In this paper a locally mass conservative finite volume method is employed to model the one-dimensional, two-phase immiscible flow in a poroelastic media. Since, an appropriate choice of primary variables is critical in simulating multiphase subsurface flow, depending on such a choice, the governing equations can be expressed in different forms. By implementing Picard iteration to a highly nonlinear system of equations, three numerical models including pressure form, mixed form and mixed form with a modified Picard linearization are developed in this study. These models have been evaluated in terms of stability, convergence and mass conservation in various one-dimensional test cases. Selecting water saturation in the mixed form as a primary variable, which is not frequent in the geotechnical engineering, could produce convergence problems in transition from saturated to unsaturated regimes, but in other conditions show good convergence and also mass balance properties. The pressure form and the mixed form with a modified Picard linearization converge in all test cases even near the fully saturated conditions. The pressure form suffers from poor mass balance and the mixed form with a modified Picard linearization poses superior mass balance property than the pressure form. In order to solve the coupled multiphase flow and geomechanics, two coupling strategies are used, first the fully coupled approach and second the iterative algorithm based on the fixed-stress operator split. Comparison between the total number of iterations and the total execution time of the fully coupled method and the fixed-stress schemes are presented through different one-dimensional examples. The accuracy, robustness and efficiency of the fixed-stress method have been demonstrated due to the reduced CPU time and low values of error for different variables.

1. Introduction

The coupling between multiphase flows and geomechanics is of significant interest in a diverse range of engineering fields. In reservoir engineering, examples of applications include land subsidence, hydraulic fracturing, wellbore instability, casing damage and sand production [1]. Within the field of environmental engineering, soil contamination problems caused by the release of petroleum hydrocarbons and immiscible industrial wastes are highly nonlinear and challenging to be solved [2–4] and in some cases they may require a coupled hydro-mechanical analysis in deformable subsystems [5,6]. Moreover, consolidation of partially saturated soils and land settlement due to groundwater pumping are problems of considerable concern in soil mechanics and geotechnical engineering in which coupled simulators are needed [7–11].

To describe multiphase subsurface flow, an appropriate choice of primary variables is critical in simulating the resulting nonlinear system [12,13]. Depending on such a choice, the governing equations can be expressed in different forms. In the context of multiphase flow, the basic formulations involve the pressure and the saturation of the fluid phases. For these two types of unknowns, the formulations can be derived as: "the pressure form" in which the state variables are the fluid pressures, "the saturation form" where the saturation of the fluid phases set as primary variables and "the mixed form" in which both pressure and saturation appear as unknowns [14]. Since, it is infeasible to model the saturated regions with the saturation form of flow equations, this approach is not well adopted. Also, because of the assumption of the deforming porous media, the solid skeleton displacement is set as the third independent variable. For the mixed form, Li [15] and Li et al. [16] developed a model based on state variables including degree of water saturation, pore water pressure and solid displacement for the water–oil and water–air systems, respectively. In Ref. [17], formulations based on gas pressure, water

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saturation and displacement as primary variables have been proposed in a water and gas system. For the pressure form, in [6,9,18–20] the pressures of the wetting and non-wetting phase act as basic variables, while in [13,21–23] capillary pressure is one of the related primary variables. Comparative studies on selecting primary variables in the case of a rigid porous media has been discussed in the literature [14,24–28]. Numerical simulation based on pressure form, provides unique and continuous solution. Models of this type could be used in both unsaturated and saturated zones, but the method suffers from poor mass balance [14,24,26]. In contrast, mixed form achieves a better mass balance. Also, in [14] the mixed form of multiphase flow has been linearized with a modified Picard iteration which results in excellent mass balance accuracy.

In the cases where solid deformation is involved, due to the above mentioned advantages the pressure form has been extensively used. In [29] the mass conservation errors have been examined for the coupled geomechanics and multiphase flow for two problems in water–oil reservoir. Since, in modeling multiphase flow in a deformable porous media, most attention has been paid to stability and convergence properties and in comparison little effort has been directed to mass conservation analysis, in this research a detailed comparative study has been performed on this aspect alongside of the stability and convergence criteria.

Due to the highly nonlinear nature of the governing system of equations, numerical discretization should be implemented. Different spatial techniques have been used to solve the coupled equations. The finite element method is the most popular in soil consolidation problems and geotechnical engineering [6–9,11,13,18,20,22,23,30]. Despite advantages of this method in dealing with complex geometries and unstructured grids, numerical instabilities can occur for the standard finite element when strong pressure gradients appear [31–34]. The other numerical method which is widely used in the reservoir problems is finite volume method (FVM). This computational scheme preserves local conservation and is capable of capturing more accurate solution for heterogeneous material and especially at the discontinuities, as illustrated in [35]. In [31], the finite volume method has been employed for discretization of the two-phase flow and the nodal based finite element scheme for the mechanical equation. The proposed model has been verified for the water-flooding problem in an oil reservoir. Also, in [36,37], finite volume method has been used to solve the Richards equation in a rigid soil. Because of the advantages of the FVM in local conservation at the element level and eliminating pressure oscillations, this approach has been implemented in this study to solve the coupled hydro-mechanical problem.

In order to solve the hydro-mechanical coupled set of equations, two strategies can be used, first the fully coupled approach and second the sequential algorithm [35,38–40]. There are several types of sequential methods based on the different degrees of coupling which can be categorized into iteratively, explicit and loosely coupled schemes [35,38–40]. In a fully coupled fashion, one matrix system is built to solve simultaneously the equilibrium equation and the continuity equations for the immiscible flowing fluids [17,30,41–44]. Despite the stability and convergence properties of this scheme, computational cost is the issue which may make the algorithm inefficient. In contrast, by using sequential strategies, computational speed will improve, while accuracy, stability and convergence properties are affected. Among the sequential schemes, the iteratively coupled with tight convergence criteria provide higher accuracy which also has the flexibility and modularity properties of the staggered schemes [29,35,45,46]. In this approach, the coupled system of equations is split into two subproblems, which are the geomechanical equilibrium and the mass balance equations of the fluid phases. The data exchange is performed iteratively between these two portions at each time step until convergence is achieved. To overcome convergence problems which the sequential schemes deal with, different operator splits, namely, drained, undrained, fixed-strain and fixed-stress splits, have been proposed [35,46–48]. Stability analysis indicates that among these operator splits, fixed-stress split is unconditionally stable for the backward Euler time discretization, even in an incompressible system [31,46,48] and takes less number of iterations to converge [35,48]. In this method the flow equations are solved first by freezing the time-variation of the volumetric stress [31,46,48]. This method has been addressed in this paper to solve the coupled system of equations. The other sequential schemes including explicit and loosely coupled suffer from low accuracy [35] and are not included in this research.

The objectives of this study are to compare different forms of the multiphase flow in poroelastic media including pressure form, mixed form and mixed form with a modified Picard linearization in terms of stability, convergence and mass conservation, in the context of FVM. Moreover, two coupling methods of fully coupled and iteratively coupled are presented for the coupled multiphase flow and geomechanics in the presence of capillarity and the accuracy and efficiency of these two schemes are analyzed. To authors' knowledge this work is the first systematic comparative study of mathematical formulations and numerical coupling strategies for two-phase flow in deforming porous media.

Mathematical models for coupled multiphase flow and geomechanics, based on the different sets of primary variables are derived in Section 2. In Section 3, the finite volume formulations of different forms of the governing equations are generated and then, the coupling strategies are illustrated. Numerical results and comparisons are presented in Section 4, and some conclusions are drawn at the end.

2. Mathematical formulations and governing equations

The full dynamic behavior of multiphase systems based on averaging theories and a classical point of view on Biot's theory is developed in [7]. Since throughout this paper the numerical solution of the resulting governing equations is dealt with, the mathematical model using Biot's theory has been presented. In this physical approach, the mass balance equation for the solid phase can be written as [7]

$$\frac{\partial(1-n)\rho_s}{\partial t} + div((1-n)\rho_s \mathbf{v}_s) = 0 \tag{1}$$

where n is the porosity of the medium, ρ_s is the density of the solid phase, t is time and $\mathbf{v}_s$ is the solid phase velocity. Also the mass conservation equation for each fluid phase can be expressed as follows

$$\frac{\partial(nS_\alpha\rho_\alpha)}{\partial t} + div(nS_\alpha\rho_\alpha\mathbf{v}_\alpha) = 0 \tag{2}$$

where S_α, ρ_α and $\mathbf{v}_\alpha$ are the degree of saturation, density and absolute velocity of the fluid phase α, respectively. To formulate final forms of continuity equations, the time derivative of the above equations are expanded and then Eq. (1) divided by ρ_s, is added to Eq. (2) divided by $S_\alpha\rho_\alpha$. By introducing relative velocities of flowing phases with respect to the solid phase as $\mathbf{v}_{\alpha s} = \mathbf{v}_\alpha - \mathbf{v}_s$ and the material time derivative as $\frac{D(*)}{Dt} = \frac{\partial(*)}{\partial t} + div(*).\ \mathbf{v}_s$, we have [7]:

$$\frac{(1-n)}{\rho_s}\frac{D\rho_s}{Dt} + div\ \mathbf{v}_s + \frac{n}{\rho_\alpha}\frac{D\rho_\alpha}{Dt} + \frac{n}{S_\alpha}\frac{DS_\alpha}{Dt} + \frac{1}{S_\alpha\rho_\alpha}div(nS_\alpha\rho_\alpha\mathbf{v}_{\alpha s}) = 0 \tag{3}$$

By considering the solid and fluid phases as compressible, constitutive relationships for the material time derivatives of the densities of these phases are needed for the case of isothermal condition.