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Research Paper

Sliding and damage criteria investigation of a micromechanical damage model for closed frictional microcracks

H. Molladavoodi *

Department of Mining and Metallurgical Engineering, Amirkabir University of Technology, Tehran, Iran

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ABSTRACT

In this paper, a micromechanical damage model formulation for closed frictional microcracks considering the coupling between frictional sliding and damage evolution was reviewed. Then, the sliding criterion for closed microcracks was modified to consider the matrix containment of sliding on microcracks. Then, the adjusted model was programmed as a constitutive model to simulate the UCS test. On the other hand, some different forms of damage criteria have been proposed in phenomenological way in literature for micromechanical damage models. The effect of damage criterion on simulated sample behavior was studied with a variation of damage criterion input parameter. The exponential and tangential damage criteria are capable to control the hardening and softening behavior. Furthermore, the influence of matrix containment of sliding on microcracks faces on the simulated sample behavior was studied in this paper. According to the numerical results, the calculated strength of the simulated sample is sensitive to the matrix resistance parameter against sliding on microcracks. The simulation results are in agreement with the experimental data.

1. Introduction

The rock materials demonstrate nonlinear mechanical response and irreversible behavior in particular under high compressive stress states. Under compressive stress fields, frictional sliding on microcracks and damage induced by microcracks are both important mechanisms of inelastic deformations in rock material. To simulate the coupled plastic deformation and induced damage, phenomenological damage models have been first proposed in the framework of irreversible thermodynamics, for instance Dragon and Mroz [1], Ju [2], Hayakawa and Murakami [3], Chiarelli et al. [4], Shao et al. [5], Voyiadjis et al. [6], Mortazavi & Molladavoodi [7] and others. In spite of its easily formulation and implementation, there are some concepts not based on physical mechanisms at microscopic scale. Whereas the microcracked rock can be considered as a heterogeneous composite with a matrix weakened by microcracks. In the micromechanical damage model, a micro to macro transition called homogenization or up-scaling methods lead to evaluate the overall (effective) elastic properties. After that, microstructure evolution such as the growth of microcracks which gives rise to a reduction of macroscopic stiffness or strength and inelastic deformation is determined by damage evolution rule [8]. The micromechanical damage models first have been formulated for open and frictionless microcracks condition [9,10]. Zhu et. al. [11,12] developed a micromechanical damage model for closed microcracks state under compressive stress fields. Under closed microcracks condition, Zhu et. al. [11–13] & Xie et. al. [14] considered coupling between frictional sliding on microcracks faces and microcrack growth (damage evolution). Zhu et. al. [11–13] compared capability of different homogenization schemes to simulate semi-brittle material behavior.

In this paper, this formulation was reviewed briefly. Then, the sliding criterion of microcracks was modified in order to consider the matrix containment of sliding on microcracks. Next, the modified micromechanical damage model was programmed in C++ environment and implemented into DEM code to simulate the compression strength test. On the other hand, there are some different forms of damage criteria proposed in phenomenological way in literature for micromechanical damage models. The qualitative capabilities of damage criteria for micromechanical damage models to simulate the rock material behavior were studied in this paper.

Throughout the paper, the following notation on dyad product of any second-order tensors $\underline{\underline{A}}$ and $\underline{\underline{B}}$ will be used: $\left(\underline{\underline{A}} \otimes \underline{\underline{B}}\right) = A_{ij}B_{kl}$. Double dot product is defined on fourth-order

* Tel.: +98 21 64542942; fax: +98 21 6405846.
E-mail address: hamedavodi@aut.ac.ir

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Nomenclature

$\mathbb{A}$	fourth-order concentration tensor
$\mathbb{C}^{hom}$	macroscopic and homogenized stiffness tensor
φ^c	volume fraction
$\mathbb{C}^s$	stiffness tensor of solid matric
$\mathbb{C}^c$	stiffness tensor of microcrack
$\mathbb{P}_\epsilon$	fourth order Hill tensor
$\mathbb{P}_d$	fourth order inclusion distribution tensor
k^s	matrix bulk modulus
μ^s	matrix shear modulus
$\mathcal{N}$	microcracks density
d	damage variable
$[\underline{u}]$	local displacement discontinuity
$\underline{\underline{E}}^c$	total macroscopic inelastic strain tensor
β	opening-closing
$\underline{\gamma}$	shear sliding vector
W	free energy of the RVE
$\underline{\underline{\sigma}}^c$	local stress field applied on microcracks
$\underline{\underline{S}}^c$	deviator part of $\underline{\underline{\sigma}}^c$
p^c	mean part of $\underline{\underline{\sigma}}^c$
α	frictional parameter of the microcracks faces
F^d	thermodynamic force associated with damage
f	damage criterion
d_0	initial damage variable
d_c	critical damage variable
$\dot{\lambda}^d$	damage multiplier

tensor ($\mathbb{C}$) and second order tensor ($\underline{\underline{E}}$) as $\left(\mathbb{C} : \underline{\underline{E}}\right) = \mathbb{C}_{ijkl}E_{kl}$. With the second rank identity tensor, the usually used fourth order isotropic tensors $\mathbb{I}$ and $\mathbb{J}$ are expressed in components form as $I_{ijkl} = 1/2\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)$ and $J_{ijkl} = 1/3\left(\delta_{ij}\delta_{kl}\right)$ respectively. The deviatoric operator $\mathbb{K}$ is then obtained by $\mathbb{K} = \mathbb{I} - \mathbb{J}$.

2. Homogenization principle

The rock microstructure can be described by the elasticity tensor dependent on the microscale coordinates $\mathbb{C}(\underline{z})$. A representative volume element (RVE) V occupying microcracks and having the boundary ∂V is adopted as shown in Fig. 1.

The macroscopic quantities such as $\langle\underline{\underline{\sigma}}\rangle$ and $\langle\underline{\underline{\varepsilon}}\rangle$ are defined as the volumetric average of the microscopic fields. Because of the uniqueness of the solution, the local strain tensor $\underline{\underline{\varepsilon}}$ inside the domain V depends linearly on the macroscopic uniform strain tensor $\underline{\underline{E}}$ applied on ∂V.

$$\underline{\underline{\varepsilon}} = \mathbb{A} : \underline{\underline{E}} \tag{1}$$

$\mathbb{A}$ is a fourth-order strain concentration tensor. The constitutive relation for each phase is given in the form $\underline{\underline{\sigma}}(z) = \mathbb{C}(z) : \underline{\underline{\varepsilon}}(z)$. By averaging of the microscopic stress and using Eq. (1), the macroscopic and homogenized stiffness tensor can be explained as

$$\mathbb{C}^{hom} = \langle\mathbb{C}(z) : \mathbb{A}\rangle \tag{2}$$

Inhomogeneous material can be described by an equivalent homogenous material. Based on Eshelby solution [15] of equivalent homogenous material, the concentration tensor of each phase ($\mathbb{A}^c$) is constant. Therefore, the stiffness tensor of the cracked media can be expressed as

$$\mathbb{C}^{hom} = \mathbb{C}^s + \sum_{c=1}^{N}\varphi^c(\mathbb{C}^c - \mathbb{C}^s) : \mathbb{A}^c \tag{3}$$

where φ^c and $\mathbb{A}^c$ are respectively the volume fraction and the concentration tensor of the rth microcrack family. The RVE is composed

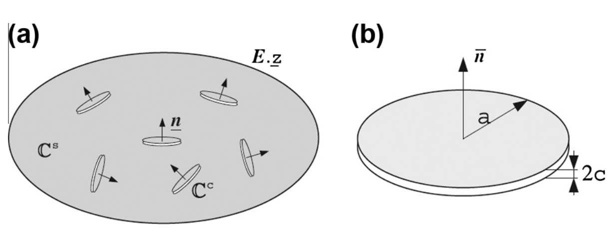

Fig. 1. (a) RVE of microcracked rock; (b) schematic representation of a penny-shaped microcrack [13].

of an isotropic linear elastic matrix with stiffness tensor $\mathbb{C}^s$ and of a random distribution of microcracks with stiffness tensor $\mathbb{C}^c(\mathbb{C}^c = 0$ in its opening state). To evaluate the homogenized stiffness tensor ($\mathbb{C}^{hom}$), the microcrack concentration tensor ($\mathbb{A}^c$) must be determined. It is generally used from the Eshelby solution [15] and analytical schemes such as dilute, Mori and Tanaka [16], Ponte-Castaneda and Willis [17] schemes to evaluate the microcrack concentration tensor $\mathbb{A}^c$ in Eq. (3).

The Ponte-Castaneda and Willis [17] scheme can consider both effects of the shape and spatial distribution of microcracks rather than other schemes. Among the considered homogenization schemes, only the PC-W one has the ability of properly taking into account the influences of interactions between microcracks [13]. In analytical schemes based on Eshelby solution, there is the fourth order Hill tensor $\mathbb{P}_\epsilon$ to define the Eshelby tensor $\mathbb{S}_\epsilon = \mathbb{P}_\epsilon : \mathbb{C}^s$. However, In Ponte-Castaneda and Willis [17] scheme, there is another fourth order tensor ($\mathbb{P}_d$) to consider the inclusion distribution. For a spherical distribution of inclusion $\mathbb{P}_d$ is [13,17]

$$\mathbb{P}_d = \frac{\alpha_1}{3k^s}\mathbb{J} + \frac{\alpha_2}{2\mu^s}\mathbb{K} \quad \text{with} \quad \alpha_1 = \frac{3k^s}{3k^s + 4\mu^s}$$
$$\alpha_2 = \frac{6(k^s + 2\mu^s)}{5(3k^s + 4\mu^s)} \tag{4}$$

k^s and μ^s are the bulk and shear modulus of the matrix respectively. When all microcracks have the same shape, the homogenized stiffness tensor can be explained as [17]

$$\mathbb{C}^{hom} = \mathbb{C}^s + \left[\mathbb{I} - \mathbb{T} : \mathbb{P}_d\right]^{-1} : \mathbb{T} \quad \text{with } \mathbb{T} = \varphi\left[(\mathbb{C}^c - \mathbb{C}^s)^{-1} + \mathbb{P}_\epsilon\right]^{-1} \tag{5}$$

2.1. Damage variable

On the assumption that the RVE of the rock material consists of penny shaped flat microcracks and solid matrix. Also the microcracks are randomly distributed in the solid matrix. According to Fig. 1, each microcrack looks like a flat elliptic with radius a, half opening c, and opening to length aspect ratio of microcrack ($\epsilon = \frac{c}{a}$). The volume fraction of microcracks in RVE can be taken from

$$\varphi = \frac{4}{3}\pi a^2 c\mathcal{N} = \frac{4}{3}\pi\epsilon d, \quad d = \mathcal{N}a^3 \tag{6}$$

$\mathcal{N}$ is the microcracks density of RVE. d is the damage variable used in micromechanical models. For open microcracks ($\mathbb{C}^c = 0$), by substituting φ from Eq. (6) into Eq. (5), $\mathbb{T}$ takes as the following form