Computers and Geotechnics 67 (2015) 229–238

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Research Paper

Influence of embedment, self-weight and anisotropy on bearing capacity reliability using the random finite element method

J.M. Pieczyńska-Kozłowska [a,*], W. Puła [a], D.V. Griffiths [b], G.A. Fenton [c]

[a] Department of Civil Engineering, Wrocław University of Technology, Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland
[b] Department of Civil & Environmental Engineering, Colorado School of Mines, 1500 Illinois Street, Golden, CO 80401, USA
[c] Dalhousie University, 1360 Barrington Street, PO Box 15000, Halifax, NS B3H 4R2, Canada

ARTICLE INFO

Article history:
Received 24 July 2014
Received in revised form 30 January 2015
Accepted 22 February 2015
Available online 3 April 2015

Keywords:
Random finite element method
Bearing capacity
Strip foundation
Anisotropic random field

ABSTRACT

The paper expands on previous work by the authors on bearing capacity of random $c' - \phi'$ soils using the random finite element method. The refinements in the present work include the influence of embedment, soil self-weight and anisotropy which were not considered previously. The study focuses on a grey-blue clay from Taranto in Italy, for which stochastic strength parameters were well documented. Results show that the influences of embedment, self-weight and anisotropy can be significant and lead to more realistic estimates of bearing capacity reliability. Finally a probability distribution of the bearing capacity was estimated and used to calibrate safety factors for reliability purposes.

1. Introduction

An increasing number of studies in the field of geotechnical engineering are considering the random character of subsoil when analysing the bearing capacity of a shallow foundation. This issue is important because it allows for the optimisation of structures that interact with soil, which are often a substantial part of the design and construction processes. The basic approach is to model the random variability of soil properties. Several papers on this subject were published in the 1960s and early 1970s, including the pioneering works of Lumb [21,22] and Schultze [29]. With the development of numerical methods and the increasing computational ability of computers, techniques for including the spatial variability of soil have been developed, leading to the modelling of soil parameters via random fields. The concepts of random functions and random fields appeared in papers by Lumb [23] and Alonso and Krizek [3]. In particular, the work of Vanmarcke [31–33] was of great importance to subsequent research. Applications of random fields in the modelling of soil parameters grew significantly following the work of Fenton and Vanmarcke [10], who developed methods of generating random fields with the use of the local average subdivision (LAS). Similar concepts have appeared in the modelling of geological deposits, especially in the work of Krige [19,20] and Zubrzycki [34]. It was only after random field concepts were combined with advanced finite element analysis [16] that meaningful practical geotechnical applications became feasible.

Griffiths and Fenton [15], Fenton and Griffiths [11] and Fenton et al. [14] studied the bearing capacity of a shallow foundation on soil with random characteristics. The analyses were performed using the random finite element method (RFEM), whose algorithms are a combination of random field theory, the classical finite element method (FEM) and Monte Carlo simulations (e.g., Fenton and Griffiths [13]). A special feature of this algorithm is that it uses multiple repetitions of calculations for different realisations of the random field. Thus, both the expected value of the random resistance from the realisation set and the probability distribution can be estimated with the use of Monte Carlo simulations and the resulting large number of realisations.

In recent years there have been a few modelling studies combining random fields with FEM for bearing capacity evaluations. In the paper by Kasama and Whittle [18] the bearing capacity of undrained soil was investigated using numerical limit analysis. In this study the undrained shear strength was modelled as an isotropic random field. The paper by Rahman and Nguyen [27] also considered undrained bearing capacity but expanded the analyses to include anisotropic random field modelled by Local Average Subdivision Method. Cassidy et al. [6] considered combined loading of strip footings subjected to vertical, horizontal and moment

* Corresponding author.
E-mail addresses: joanna.pieczynska-kozlowska@pwr.edu.pl (J.M. Pieczyńska-Kozłowska), wojciech.pula@pwr.edu.pl (W. Puła), d.v.griffiths@mines.edu (D.V. Griffiths), gordon.fenton@dal.ca (G.A. Fenton).

http://dx.doi.org/10.1016/j.compgeo.2015.02.013
0266-352X/

Nomenclature

c	cohesion
ϕ	friction angle
v	Poisson ratio
E	Young's modulus
γ	soil unit weight
π	PI
B	footing width
D	footing high of embankment
$q_f = q_{\text{ult}}$	ultimate bearing capacity
q_{FEM}	bearing capacity value computed by finite element method
q_d	bearing capacity design value
N_c	bearing capacity factor – cohesion
N_γ	bearing capacity factor – unit weight
N_q	bearing capacity factor – overburden pressure
p_f	probability of failure
β	reliability index
Φ	cumulative distribution function of the standard distribution
x_i	single realisation of random variable x
$f(x)$	probability density function
$E[X]$	expected value
Var[X]	variance
ρ	correlation function
τ	separation vector absolute distance between points in field
γ^{VAR}	Variance reduction function
$\mu_{\text{ln}x}$	mean value in underlying normal distribution
μ_c	mean value of cohesion
μ_ϕ	mean value of friction angle
μ_x	mean value in lognormal distribution
μ_{qf}	mean value of bearing capacity
σ_x	standard deviation in lognormal distribution
$\sigma_{\text{ln}x}$	standard deviation in underlying normal distribution
σ_c	standard deviation of cohesion
σ_ϕ	standard deviation of friction angle
σ_{qf}	standard deviation of bearing capacity
COVc	coefficient of variation of cohesion
COVϕ	coefficient of variation of friction angle
COVq_f	coefficient of variation of bearing capacity
θ	fluctuation scale, correlation length
θ_c	fluctuation scale of cohesion
θ_ϕ	fluctuation scale of friction angle
θ_x	horizontal fluctuation scale
θ_y	vertical fluctuation scale
θ/B	fluctuation scale normalised on footing width
$G_{\text{ln}c}$	Gaussian random field of cohesion
G_ϕ	Gaussian random field of friction angle
FS	factor safety

loads. It is worth mentioning new simulation approaches recently proposed by Ahmed and Soubra [1] and by Al-Bittar and Soubra [2]. Fenton and Griffiths [11] analysed the bearing capacity of a surface footings on $c' - \phi'$ soils, i.e., soils with no footing embedment. In that study, the soil unit weight was ignored and the random fields of cohesion and the internal friction angle were assumed to be isotropic, i.e., the spatial correlation was the same in both the vertical and horizontal directions.

This article is a supplement to and continuation of the aforementioned work. The algorithm used by Fenton and Griffiths [11] was expanded to consider foundation embedment and soil unit weight. In addition, the influence of anisotropic random fields and the cross-correlation between soil parameters was investigated.

2. Assumptions used in the computations

The traditional computational approach of estimating the bearing capacity under drained conditions presupposes the designation of the bearing capacity with the use of the Terzaghi equation [30], which is based on the mechanism described by Prandtl [25]:

$$q_f = c'N_c + qN_q + \frac{1}{2}\gamma B N_\gamma \tag{1}$$

where q_f is the bearing capacity, c' is the cohesion, q is the overburden pressure, γ is the soil unit weight, B is the footing width, and N_c, N_q and N_γ are the bearing capacity factors, which depend on the internal friction angle, ϕ'. The study by Fenton and Griffiths [11], using the RFEM, only considered the component of Eq. (1) associated with the cohesion, i.e.,

$$q_f = c'N_c \tag{2}$$

in which the bearing capacity factor, N_c, is given by Ref. [25]:

$$N_c = \frac{\exp(\pi \tan\phi')\tan^2\left(\frac{\pi}{4} + \frac{\phi'}{2}\right) - 1}{\tan\phi'} \tag{3}$$

The current study includes all elements of Eq. (1) while also considering the influences of different effects of random soil resistance. The computational scheme of the study is summarised in Fig. 1.

Soil parameters were modelled as random fields characterised by probability distributions and a specified correlation structure. The soil analysed was Taranto Blue Clay, whose properties have been described by Cafaro et al. [4], Cafaro and Cherubini [5], Cherubini [8] and Cherubini et al. [9]. The statistical data were obtained using in situ tests, including CPT penetration tests.

Each test result is described using a trend function with the parameters corresponding to the mean value and residual variance of the parameter around the trend. The results are presented in Table 1.

The fluctuations of these values were then modelled using random fields with a zero mean and unit standard deviation. The correlation structure was isotropic and characterised by the vertical values of the fluctuation scale [31], which was measured by Cafaro and Cherubini [5]. Weak (or wide-sense) stationarity of

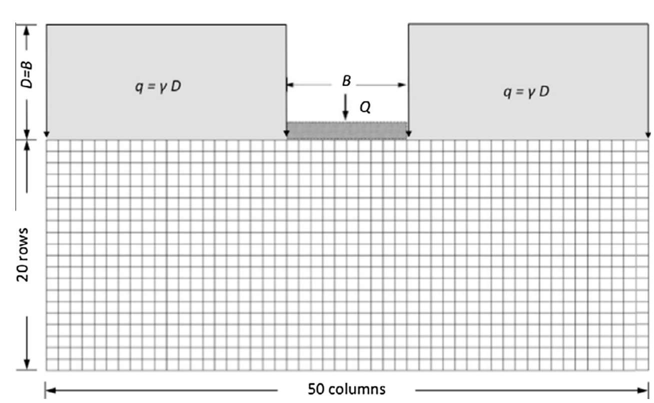

Fig. 1. Computational scheme used in the analysis.