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Analytical solutions for nonlinear consolidation of soft soil around a shield tunnel with idealized sealing linings

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A B S T R A C T

This paper presents the analytical solutions for nonlinear consolidation of soft soil around a shield tunnel with idealized sealing linings. By introducing the empirical relation between permeability and compressibility, along with the conformal transformation, the governing equations of nonlinear consolidation are established, and the corresponding analytical solutions are derived. Then, the Terzaghi consolidation solutions are derived from the degenerate governing equation of nonlinear consolidation. Through the predictions of different consolidation theories in both completely permeable and impermeable lining conditions, the influences of a tunnel acting as a drain and impacting the dissipation of pore pressure, degree of consolidation, long-term ground settlements and ground settlement rates are investigated. During the early stages of consolidation, the case studies reveal that the predictions made by this study strongly agree with the field data when a completely permeable lining is applied. This study confirms that a tunnel acting as a drain can accelerate the consolidation of soil and enlarge soil deformation due to consolidation. During long term consolidation, a notable nonlinearity of the soil consolidation is exhibited by a small and gradually decreasing settlement rate, showing agreement with the tendency of field data from the impermeable conditions.

1. Introduction

Many theoretical and field studies have been performed on tunneling induced ground settlement, yet most of these studies are concerned with short-term ground movement induced by tunnel construction [1]. However, the ground settlement around tunnels in soft clays continuously increases for many years after construction, especially around those built in coastal areas. Peck proposed an empirical formula for tunneling settlement [2], and then, Fang et al. proposed a time dependent equation for ground settlement above a tunnel's center line [3]. Because of the large difference between the hydraulic head inside and outside tunnel linings after the construction of a tunnel, as well as joint leaking caused by segmented construction and differential settlement, Wongsaroj et al. concluded that tunnels in soft clays actually act as drains [4], which introduces a new drainage boundary condition that leads to long-term reductions in pore pressure and associated consolidation settlements. Mair and Taylor argued that the resulting settlement profile at the ground surface will tend to be considerably wider than the profile associated with construction [5].

Further research has confirmed, through measurements of pore pressures around tunnels, that tunnels constructed in soils with low-permeability act as drainage boundaries [6,7]. Moreover, various case studies and numerical analyses reveal that segmentally lined tunnels still act as drains when grouted [8,9]. Since then, many studies on this problem have been performed by using analytical and numerical tools and by considering the lining as a homogeneous permeable body [10–16]. However, the nonlinear behavior of soil in long-term consolidation has not been taken into consideration by the aforementioned studies.

Based on an enormous amount of field measurements and theoretical research work, Shirlaw concluded that the proportion of long-term consolidation induced by construction disturbance to total settlement is between 30% and 90% [17]. Considering the immediately occurring construction settlement, the proportion of consolidation to total settlement after tunnel operation is even higher. Duncan found that the permeability and compressibility of soft clays change very much during long-term consolidation, causing a serious deviation from using the conventional consolidation theory in predicting the long-term settlement of soft clays [18]. As an effective tool to overcome the limitations of the conventional consolidation theory, the nonlinear consolidation theory can well predict the nonlinear behavior of soft clays [19–22].

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This paper idealizes the linings of tunnels as being completely permeable or completely impermeable, which represents the upper and lower limits of the induced soil consolidation behavior, and investigates how a tunnel acting as a drain affects the long-term behavior of the surrounding soil. In this paper the tunnel support is assumed to be rigid, so that the ground deformation around the tunnel section is short-term behavior compared with the effects of nonlinear consolidation. It is assumed that most of the ground deformation around the tunnel section is already completed during tunneling. Thus the tunnel support is not taken into account in the analysis presented. By introducing the classic *e*-lg *k* relation of nonlinear consolidation, and the conformal transformation, the corresponding consolidation equation is established. Then, the analytical solutions are developed for a long-term nonlinear consolidation with idealized drainage boundaries. After that, the obtained nonlinear solutions for excess pore pressure, degree of consolidation, and ground settlements are discussed by comparing them with the conventional Terzaghi consolidation solution corresponding to these special cases. Through comparison with settlement curves of field measured data and a modified Peck formula, the rationality of the upper and lower limits by the presented analytical solutions of both idealized lining conditions is discussed.

2. Statement of the problem

The geometry of a circular tunnel constructed in soft clay with an idealized lining is shown in Fig. 1. The radius of the tunnel buried at a depth '*h*' in soft clay is *r*, and the origin of the coordinate is at the ground surface above the tunnel. Considering Davis's hypothesis for nonlinear consolidation of soil [19], the basic assumptions made in developing the solutions presented in this study are stated below:

1. The longitudinal length of the tunnel is large enough to meet plane strain conditions.
2. The soil around the tunnel is regarded as a half space with a cavity.
3. The soil around the tunnel is an isotropic saturated porous medium, and the permeability and compressibility coefficients are subjected to the *e*-lg *k* and *e*-lg σ relation, respectively, during the consolidation.
4. Soil particles and pore water are incompressible. Darcy's law is valid. The excess pore pressure at the ground surface and at an infinite depth is zero.
5. Soil deformation during consolidation is small, neglecting the effect on the coordinate system.

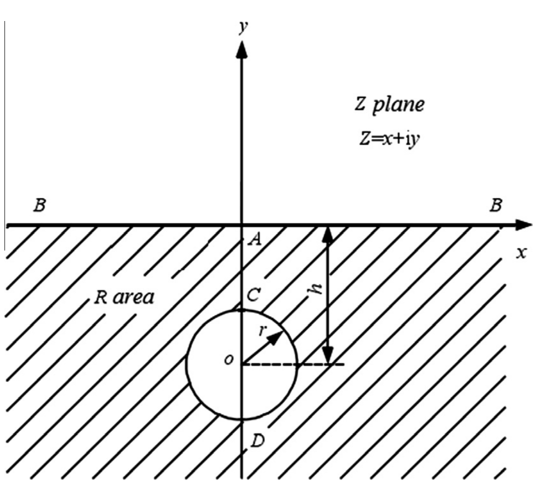

Fig. 1. Geometry of a tunnel in soft clay.

6. Soil deforms freely, neglecting the arch effect of soil itself and the tunnel.
7. The construction of a shield tunnel only causes excess pore pressure, and does not change the distribution of total vertical stress.

3. Solutions for nonlinear consolidation of soft clay around tunnel

3.1. Governing equation

Based on basic assumption (3), the permeability and compressibility coefficients of the soil are respectively subjected to an empirical nonlinear relation by Davis, Bardon, Mesri, and Xie [19–22], as below:

$$e = e_0 - C_c \lg\left(\frac{\sigma'}{\sigma'_0}\right) \tag{1}$$

$$e = e_0 + C_k \lg\left(\frac{k_s}{k_{s0}}\right) \tag{2}$$

where e is the void ratio, e_0 is the initial void ratio, σ'_0 is the initial effective stress, σ' is the effective stress, k_{s0} is the initial permeability coefficient of the soil, k_s is the permeability coefficient of the soil, C_c is the initial compression index, and C_k is the initial permeability index.

Then, from Eqs. (1) and (2), it can be obtained that

$$k_s = k_{s0}\left(\sigma'_0/\sigma'\right)^{\frac{C_c}{C_k}} \tag{3}$$

$$m_v = -\frac{1}{1+e_0}\frac{\partial e}{\partial \sigma'} = m_{v0}\frac{\sigma'_0}{\sigma'} \tag{4}$$

where m_v is the coefficient of compressibility, and $m_{v0} = C_c/(1+e_0)\sigma'_0 \ln 10$ is the initial coefficient of compressibility.

Then, substituting m_{v0} into Eq. (4), we can obtain

$$m_v = \frac{C_c}{(1+e_0)\ln 10}\frac{1}{\sigma'} \tag{5}$$

From the free strain assumption, the continuity equation of saturated soil can be derived

$$\frac{1}{\gamma_w}\frac{\partial}{\partial x}\left(k_s\frac{\partial \Delta u}{\partial x}\right) + \frac{1}{\gamma_w}\frac{\partial}{\partial y}\left(k_s\frac{\partial \Delta u}{\partial y}\right) = -\frac{\partial \varepsilon_v}{\partial t} \tag{6}$$

$$\frac{\partial \varepsilon_v}{\partial t} = \frac{1}{1+e_0}\frac{\partial e}{\partial t} \tag{7}$$

where γ_w is the bulk density of water, Δu is the excess pore pressure, and ε_v is the body strain.

From the principle of effective stress, we can obtain

$$\frac{\partial \sigma'}{\partial t} = -\frac{\partial \Delta u}{\partial t} \tag{8}$$

Then, substituting Eqs. (5) and (8) into Eq. (7), we can obtain

$$\frac{\partial \varepsilon_v}{\partial t} = -\frac{C_c}{(1+e_0)\ln 10}\frac{1}{\sigma'}\frac{\partial \Delta u}{\partial t} \tag{9}$$

Assuming that the compressibility and permeability are decreasing all along with increasing pressure, (as done by Davis) [19], i.e., $C_c/C_k = 1$, by substituting Eqs. (3) and (9) into Eq. (6), the governing equation is then derived as

$$\frac{1}{\gamma_w}\frac{\partial}{\partial x}\left(k_{s0}\left(\frac{\sigma'_0}{\sigma'}\right)\frac{\partial \Delta u}{\partial x}\right) + \frac{1}{\gamma_w}\frac{\partial}{\partial y}\left(k_{s0}\left(\frac{\sigma'_0}{\sigma'}\right)\frac{\partial \Delta u}{\partial y}\right) = -\frac{C_c}{(1+e_0)\ln 10}\frac{1}{\sigma'}\frac{\partial \Delta u}{\partial t} \tag{10}$$

Using the substitution w: