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Shape optimisation of the support section of a tunnel at great depths

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ABSTRACT

The shape optimisation of a cavity is typically performed without considering the previous support, which may significantly reduce the practical significance of such analyses. Even when an excavation section is optimised, failure of the surrounding rock in a tunnel cannot be prevented in the presence of excessive in situ stress. Thus, a support should be established to protect the stability of a tunnel from the failure of the surrounding rock. This study examines the optimal shape of the support that satisfies the optimisation criterion, which minimises the largest tangential stress along the inner edge of the support, for a specific net tunnel size and support strength. The optimisation process is to solve a series of forward problems using the conformal mapping method for a plane elasticity complex function. The tangential stress along the inner edge of the support is selected as the objective function, and the coefficients of the mapping function are considered as the design variables. The minimum value of the objective function is calculated based on the mixed penalty function method and the optimal support shape that satisfies the given constraints can be obtained. The stress state of an optimally shaped tunnel support is significantly improved compared to non-optimal configurations, and the stress concentration along the inner edge of the support is minimised.

1. Introduction

Underground tunnels are widely used in hydropower, traffic, mining, and military facilities. The stability of a tunnel is closely related to the shape of the excavation section. The selection of a reasonable shape can improve the stress state in a tunnel and the self-supporting ability of the surrounding rock, which is critical for the maintenance of underground engineering.

An important indicator of an optimal excavation shape is the stress concentration at the edge of the hole [1]. In mechanics, the process of determining the optimal excavation shape of the underground cavity can be classified as a hole-shape optimisation problem. In geometry, this process is classified as a type of inverse problem, i.e., the optimal excavation shape of a tunnel is determined based on the premise that the stress satisfies certain requirements. In this process, the excavation shape is unknown. Reducing the stress concentration along the edge of the cavity is an important issue when optimising the shape of a hole in a plane. This problem is typically solved using the finite element method [2–4], finite difference method [5,6], or complex function method [7–11]. The finite element method and finite difference method have not been extensively used due to their high computational requirements. The complex function method is an analytical method that has gained popularity due to its high efficiency in stress analyses of underground tunnel problems. In the shape optimisation method that employs the complex function method, some coefficients of the mapping function serve as design variables. The first step to optimising the shape of the hole is to select the optimal criterion, as different shapes can be obtained for different optimisation criteria and different stress distributions exist along the edge of the hole. Thus, the appropriate optimisation criterion should minimise the stress concentration at the edge of the hole.

In 1976, Bjorkman et al. proposed the concept of a harmonic hole [7]. Optimal shapes for different types of acting loads were obtained based on the premise that the first invariant of the stress remains constant regardless of the existence of a hole [7,8]. However, this criterion is not practical, e.g., for specific types of loads, the shape of a harmonic hole is not practical or does not exist. In 1981, Dhir proposed the optimisation criterion to minimise the integral value of the square of the tangential stress along the edge [9]. The optimisation problem for any type of hole was solved using the complex function method, which is similar to the approach by Bjorkman. Cristescu solved a class of optimisation problems with a rounded rectangle hole [1]. The aforementioned criteria can sufficiently distribute stresses along the edge but may not necessarily yield the minimum stress concentration.

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In 1996, Lu proposed a new optimisation criterion [10]. In his opinion, the optimal shape is obtained when the maximum absolute value of tangential stress along the edge of the hole reaches the minimum stress concentration. Based on this definition, the maximum absolute value of the tangential stress is the minimum stress concentration on the edge of the hole obtained by this criterion. The optimal shapes of the tunnel excavation in Ref. [11] were based on the criterion. The optimisation in Ref. [11] is based on the premise that no tensile stress appears in the vicinity of the tunnel, as the compressive strength of rock is significantly higher than its tensile strength. To ensure that the cavity shape obtained by the criterion is acceptable in engineering practice, the size and shape of the tunnel should satisfy certain basic requirements, i.e., the width and height of the tunnel.

The optimal hole can ensure a minimum or low stress concentration along the boundary. Another question that emerges is whether there are any other points in the domain for which the stresses are larger than the stresses along the inner edge. The answer is no, which can be proven by the maximum principles for differential equations. The maximum tangential stress must be located along the boundary [11].

The shape optimisation discussions in Refs. [1–11] are based on a bare hole, i.e., they do not consider the support of the tunnel. However, shape optimisation efforts that do not consider tunnel support lack practical significance because failure of the surrounding rock in a tunnel can occur even when the using the optimal excavation section. Thus, a support should be established to protect a tunnel from the failure of the surrounding rock. The support can provide a radial supporting force on the surrounding rock; in this manner, the stress field in the surrounding rock changes, and the tangential stress along the inner edge of tunnel can be reduced.

The problem discussed in this paper is how to determine the optimal shape when the closed concrete support is established. The following assumptions are made: (1) the surrounding rock mass and lining are always linear elastic under the action of in situ stresses and their interactions and (2) because the tunnel is assumed sufficiently deep, the problem can be simplified as an infinite domain problem.

2. Theories for determining the optimal support section

2.1. Solution of the forward problem

According to [7–10], the process for obtaining the optimal shape of a hole requires solving a series of forward problems, although the optimisation of the shape of a hole is an inverse geometry problem. From the subsequent analysis, we learn that the process for determining the optimal shape of the support is equivalent to the process for solving a series of forward problems. In solid mechanics, a forward problem involves determining the stress distributions and law of deformation for an object by knowing the geometry, material properties, and external load.

In this paper, the conformal mapping method of a complex function is employed to solve the forward problem. The complicated support section (see Fig. 1(a)) in the z plane is transformed to the annular region (see Fig. 1(b)) in the ζ plane with an outer radius and inner radius of 1 and R_0, respectively, using the mapping function $z = \omega(\zeta)$. All coefficients in the mapping function can be determined when the shape of the tunnel cross-section and support thickness R_0 are known. That is, if the mapping function is described as

$$z = \omega(\zeta) = R\left(\zeta + \sum_{k=0}^{n} c_k \zeta^{-k}\right) \tag{1}$$

where R and C_k ($k = 1,\ldots,n$) can be solved by the method described in Ref. [11]. When the external loads are applied at infinity (the initial rock stress) (Fig. 1(a)) and the shapes of the tunnel and lining are symmetrical about the x axis, R and C_k ($k = 1,\ldots,n$) in Eq. (1) must be real numbers, which is the case discussed in this paper. The stress fields in the surrounding rock and support are also symmetrical about the x axis; thus, the angle θ is only discussed in $[0, \pi]$.

To solve the forward problem using the conformal transformation method, a complex function is used to solve three groups of analytic functions: $\varphi_1(\zeta)$, $\psi_1(\zeta)$; $\varphi_2(\zeta)$, $\psi_2(\zeta)$, and $\varphi_3(\zeta)$, $\psi_3(\zeta)$.

$\varphi_1(\zeta)$ and $\psi_1(\zeta)$ are two analytic functions in the outer region of the unit circle γ_2 when the tunnel is unsupported. The specific form of $\varphi_1(\zeta)$, $\psi_1(\zeta)$ can be obtained given P, Q, R, R_0, and C_k ($k = 1,\ldots,n$).

$\varphi_2(\zeta)$ and $\psi_2(\zeta)$ are two analytic functions in the outer region of unit circle γ_2 with only the lining supported; their forms can be expressed as

$$\varphi_2(\zeta) = b_0 + \sum_{k=1}^{\infty} b_k \zeta^{-k} \tag{2}$$

$$\psi_2(\zeta) = d_0 + \sum_{k=1}^{\infty} d_k \zeta^{-k} \tag{3}$$

$\varphi_3(\zeta)$ and $\psi_3(\zeta)$ are the analytic functions in the ring-shaped region that correspond to the lining after the lining is applied; their forms can be expressed as

$$\varphi_3(\zeta) = p_0 + \sum_{k=1}^{\infty} e_k \zeta^{-k} + \sum_{k=1}^{\infty} f_k \zeta^{k} \tag{4}$$

$$\psi_3(\zeta) = q_0 + \sum_{k=1}^{\infty} g_k \zeta^{-k} + \sum_{k=1}^{\infty} h_k \zeta^{k} \tag{5}$$

where b_0, d_0, p_0, q_0, b_k, d_k, e_k, f_k, g_k, and h_k in Eqs. (2)–(5) are real constants to be determined. These coefficients can be calculated according to the stress boundary conditions along L_1 and the stress and displacement continuity conditions along L_2; thus, the analytical expressions of $\varphi_2(\zeta)$, $\psi_2(\zeta)$, $\varphi_3(\zeta)$ and $\psi_3(\zeta)$ can be obtained.

The stress components for any points in the lining can be solved according to the following two equations:

$$\sigma_\rho + \sigma_\theta = 4\mathrm{Re}[\varphi_3'(\zeta)/\omega'(\zeta)] \tag{6}$$

$$\sigma_\theta - \sigma_\rho + 2i\tau_{\rho\theta} = \frac{2\zeta^2}{\rho^2}\frac{1}{\overline{\omega'(\zeta)}}\left\{\overline{\omega(\zeta)}\frac{\varphi_3''(\zeta)\omega'(\zeta) - \varphi_3'(\zeta)\omega''(\zeta)}{[\omega'(\zeta)]^2} + \psi_3'(\zeta)\right\} \tag{7}$$

where σ_ρ, σ_θ, and $\tau_{\rho\theta}$ are the stress components in orthogonal curvilinear coordinates in the z plane.

The stress components for any points in the surrounding rock mass can be solved according to the following two equations:

$$\sigma_\rho + \sigma_\theta = 4\mathrm{Re}[\varphi'(\zeta)/\omega'(\zeta)] \tag{8}$$

$$\sigma_\theta - \sigma_\rho + 2i\tau_{\rho\theta} = \frac{2\zeta^2}{\rho^2}\frac{1}{\overline{\omega'(\zeta)}}\left\{\overline{\omega(\zeta)}\frac{\varphi'(\zeta)\omega'(\zeta) - \varphi'(\zeta)\omega''(\zeta)}{[\omega'(\zeta)]^2} + \psi'(\zeta)\right\} \tag{9}$$

where

$$\varphi(\zeta) = \Gamma\omega(\zeta) + \varphi_1(\zeta) + \varphi_2(\zeta) \tag{10}$$

$$\psi(\zeta) = \Gamma'\omega(\zeta) + \psi_1(\zeta) + \psi_2(\zeta) \tag{11}$$

where $\Gamma = P(1+\lambda)/4$, $\Gamma' = P(\lambda - 1)$, $\lambda = Q/P$. From this analysis, the stress components for any point in the surrounding rock and support can be calculated with the known external load when the displacement release coefficient η [12] and the mapping function,