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An improved Monte Carlo simulation method for discontinuity orientations based on Fisher distribution and its program implementation

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A B S T R A C T

Fisher distribution is the most commonly used probability density function for discontinuity orientations. Based on Fisher distribution, Monte Carlo simulation method for discontinuity orientations was reviewed and improved. Those orientations extending beyond the edge of an upper hemisphere projection (OEBEUHP) often have an important influence on both the mean orientation and Fisher constant *K*, thus affecting simulation results. The detailed algorithms for identifying and adjusting those OEBEUHP were developed in this paper. Based on the improved method, a program for generating discontinuity orientations and plotting their stereographic projection maps, named as **MCSDO**, was developed. Due to the aforementioned adjustment, the generated orientations by **MCSDO** are close to the original discontinuity orientations, which were mapped in field, and satisfactory. Only the original orientations and target number of generated orientations need to be input. By running the program we can directly obtain the follows: orientations of generated discontinuities, mean orientations of both original and generated discontinuities, Fisher constant *K* of both original and generated discontinuities, and stereographic projection maps of both original and generated discontinuities. **MCSDO** is a freeware designed for researchers and practicing engineers, and can be easily mastered with a little computer knowledge.

1. Introduction

"Discontinuity" is a general term denoting any separation in a rock mass having zero or low tensile strength [1]. It is the collective term for most types of joints, weak bedding planes, weak schistosity planes, weakness zones, and faults [2]. Discontinuities have significant influence on the deformation [3,4], strength [5,6], permeability [7,8], stress–strain relation [9], and the failure [10] of rock masses. Discontinuity properties to be determined mainly include orientation, size, frequency, surface geometry, genetic type, and infill material [11,12]. Discontinuities are hidden in the actual rock masses, so it is impossible to investigate all discontinuities and their properties. In addition, discontinuities are generally developed in rock masses randomly and in sets [13]. Therefore, it is widely accepted that statistical methods are utilized to quantitatively study discontinuity properties [14–19].

Statistical descriptions of orientation data are best expressed as the normal vectors of the discontinuities. Probability density functions (PDF) for discontinuity orientations are mainly: Fisher distribution [14,20–23], Bingham distribution [24,25], and Bivariate normal distribution [26]. In these distributions, Fisher distribution is most commonly used since it only uses a single parameter, the Fisher constant *K*, and is an integrable function, which makes it convenient to generate random data using Monte Carlo simulation method.

Random data following normal distribution, exponential distribution and logarithmic distribution can be directly generated by built-in functions in some softwares, such as Matlab [27], while that obeying Fisher distribution cannot be directly generated by any commercial or published software. The aim of this study is to review and improve Monte Carlo simulation method for discontinuity orientations based on Fisher distribution, and develop a user-friendly program for researchers or practicing engineers using Excel Visual Basic for Application (VBA). In order to accomplish the task, calculation methods of mean orientation and *K* for a discontinuity set were introduced, first of all. Priest [21] mentioned those orientations extending beyond the edge of an upper hemisphere

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Nomenclature

$\boldsymbol{A}$	coordinate transformation matrix
A	polar stereographic projection point of the normal vector of a discontinuity on the horizontal plane
i	discontinuity number
K	Fisher constant
(l, m, n)	direction cosines of (β, α)
(l', m', n')	direction cosines of (β', α')
MCSDO	Monte Carlo simulation for discontinuity orientations program
N	total number of discontinuities from a discontinuity set
$\boldsymbol{n_i}$	upper unit normal vector of discontinuity i
$\boldsymbol{n_m}$	direction vector of mean orientation
$\boldsymbol{n_{xyi}}$	projection of $\boldsymbol{n_i}$ on the horizontal plane
$\boldsymbol{n_{xym}}$	projection of $\boldsymbol{n_m}$ on the horizontal plane
OEBEUHP	orientations extending beyond the edge of an upper hemisphere projection
$P(\beta_1 < \beta < \beta_2)$	probability that a random orientation value make an angle of between β_1 and β_2 with the mean orientation
R	radius of the great circle
$\boldsymbol{r_N}$	resultant vector of the all normal vectors
$\lvert\boldsymbol{r_N}\rvert$	norm of $\boldsymbol{r_N}$
$\boldsymbol{r_{Nu}}$	unit vector of $\boldsymbol{r_N}$
$[t]$	times of iterations
VBA	Visual Basic for Application
x_i, y_i, and z_i	components of $\boldsymbol{n_i}$ in (x, y, z) coordinate system
x_r, y_r, and z_r	components of $\boldsymbol{r_N}$ in (x, y, z) coordinate system
x_{ru}, y_{ru}, and z_{ru}	components of $\boldsymbol{r_{Nu}}$ in (x, y, z) coordinate system
(x, y, z)	original Cartesian coordinate system
(x', y', z')	new transformed Cartesian coordinate system
(x_A, y_A)	coordinates of point A
α_m	α value corresponding to the mean orientation
β_m	β value corresponding to the mean orientation
(β, α)	polar coordinate system
(β', α')	transformed new polar coordinate system
γ_{mi}	angle of intersection between $\boldsymbol{r_{Nu}}$ and $\boldsymbol{n_i}$
γ_{m-i}	angle of intersection between $\boldsymbol{r_{Nu}}$ and $-\boldsymbol{n_i}$
δ_i	dip angle of discontinuity i
δ_m	mean dip angle of a discontinuity set
θ_i	dip direction of discontinuity i
θ_m	mean dip direction of a discontinuity set
ε	convergence criteria of the iterations
μ	random number distributed according to the uniform (0, 1) distribution

projection (OEBEUHP) should be adjusted when determining mean orientation of a discontinuity set. We found that those OEBEUHP often had an important influence on both the mean orientation and Fisher constant K, thus affecting simulation results. The detailed algorithms for identifying and adjusting those OEBEUHP were developed in this paper. In order to conveniently evaluate simulation results, plotting both original and generated orientations in stereographic projection maps were also conducted. The execution flowcharts of the developed program were detailed. Lastly, a field case was given to illustrate operation and simulation results of this program, and comparative analysis of simulation results considering and ignoring the improvement proposed in this paper was also performed.

2. Review and improvement of method

2.1. Calculation of mean orientation

Fisher distribution is a symmetric distribution about the mean orientation on a sphere [20,21], and hence the mean orientation of a discontinuity set should be calculated first.

Assume that N discontinuities were mapped in a discontinuity set. One of those was denoted as i, and its dip angle and dip direction were labeled as δ_i and θ_i, respectively; its unit upper normal vector was denoted as $\boldsymbol{n_i}$, and Cartesian coordinates of $\boldsymbol{n_i}$ were denoted as x_i, y_i, and z_i, respectively; the projection of $\boldsymbol{n_i}$ on the horizontal plane was denoted as $\boldsymbol{n_{xyi}}$. Note that: (a) the Cartesian coordinate system is a right-handed one, and positive z-axis points vertically upward, as shown in Fig. 1 and (b) the dip angle δ_i is the angle of intersection between positive z-axis and $\boldsymbol{n_i}$, and the dip direction θ_i is measured as the angle between positive y-axis (North) and $\boldsymbol{n_{xyi}}$ in a clockwise, both which are referred in the geological coordinate system, as shown in Fig. 1.

The mean orientation for a set containing a total of N discontinuities can be taken as the orientation of the resultant vector of the all normal vectors, $\boldsymbol{r_N}$, and its Cartesian components x_r, y_r, and z_r can be gotten by Refs. [21,28,29]:

$$x_r = \sum_{i=1}^{N} x_i;\ y_r = \sum_{i=1}^{N} y_i;\ z_r = \sum_{i=1}^{N} z_i \tag{1}$$

where x_i, y_i, and z_i are the Cartesian components of $\boldsymbol{n_i}$, and according to the information given in Fig. 1, they can be expressed as follows [30]:

$$x_i = \sin\delta_i \cdot \sin\theta_i;\ y_i = \sin\delta_i \cdot \cos\theta_i;\ z_i = \cos\delta_i \tag{2}$$

Then, the norm of $\boldsymbol{r_N}$, $|\boldsymbol{r_N}|$, can be determined by

$$|r_N| = \sqrt{x_r^2 + y_r^2 + z_r^2} \tag{3}$$

The Cartesian components x_{ru}, y_{ru}, and z_{ru} of unit vector $\boldsymbol{r_{Nu}}$ of $\boldsymbol{r_N}$ can be calculated by:

$$x_{ru} = \frac{x_r}{|r_N|};\ y_{ru} = \frac{y_r}{|r_N|};\ z_{ru} = \frac{z_r}{|r_N|} \tag{4}$$

The inverse forms of Eq. (2) can give the mean orientation, δ_m and θ_m, which are

$$\delta_m = \cos^{-1}|z_{ru}|;\ \theta_m = \tan^{-1}\left(\frac{x_r}{y_r}\right) \tag{5}$$

where the range of δ_m is between 0 and $\pi/2$; the range of θ_m is between 0 and 2π clockwise from positive y-axis, and the coordinate quadrant of θ_m is determined by the signs of x_r and y_r.

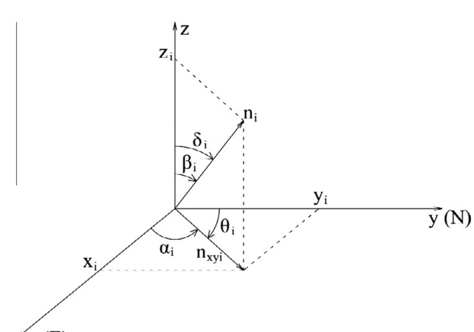

Fig. 1. A diagram to illustrate the Cartesian coordinate system (x, y, z), geological coordinate system (δ, θ), and polar coordinate system (β, α).