A coupled stress–strain and hydraulic hysteresis model for unsaturated soils: Thermodynamic analysis and model evaluation

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ABSTRACT

The thermodynamics of coupled mechanical deformation and hydraulic hysteresis in unsaturated soils is discussed, and the energy dissipation associated with the elastoplastic process and the main wetting and drying processes is derived. Based on a newly developed elastoplastic model (Hu et al., 2014) and a hysteretic water retention curve for deformable soils (Hu et al., 2013), a coupled hydromechanical model is formulated. In the coupled model, the newly defined inter-particle bonding factor from our recent work (Hu et al., 2014) plays a key role both in representing the influence of water retention curve on the mechanical behavior and in governing the evolution of the hardening force in the stress–strain response. The effect of deformation on the water retention behavior is determined according to the variation in the void ratio. It is proved that the total energy dissipation is strictly positive once plastic deformation and/or an irreversible change in the degree of saturation occurs, and therefore, our model satisfies the second law of thermodynamics. The model is validated by comparison with experimental data, and it is shown that the coupled model is able to capture the main features of coupled hydro-mechanical behavior in unsaturated soils, including hydraulic hysteresis and mechanical hysteretic behaviors during drying–wetting cycles.

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1. Introduction

Understanding coupled hydraulic hysteretic and stress–strain behavior in unsaturated soils is important for various engineering applications (e.g., [1–4]). Constitutive models are fundamental for understanding and modeling the coupled hydromechanical processes naturally occurring in soil materials. In a coupled hydromechanical model of unsaturated soil, the hydraulic model (or water retention curve) is essential because it determines the evolution of the degree of saturation. The degree of saturation has a significant effect on the internal bonding stress acting on the areas of contact between soil particles. This self-balancing bonding stress provides an additional normal stress acting to link the soil particles together [5,6] and may significantly influence the mechanical behavior. In

turn, the mechanical deformation affects the water retention behavior by changing the pore size distribution.

Thermodynamic laws provide a theoretical basis for the simulation of the coupled hydromechanical behavior of unsaturated soils. The most important contribution to the study of saturated soils, the critical-state soil mechanics [7] framework, was established in the 1960s based on thermodynamic concepts. It remains a difficult task, however, to formulate a constitutive model for the coupled hydromechanical behavior of unsaturated soils because of the difficulty in quantifying the internal bonding stress induced by water menisci and determining the evolution of the dissipation energy associated with soil skeleton deformation and hydraulic hysteresis and their coupling. Therefore, few constitutive models for unsaturated soils have been established based on thermodynamic considerations.

Alonso et al. [8] and researchers building on their work [5,9–11] provided early and important contributions. In their work, the mean net stress and suction were adopted as basic stress variables, but the influence of mechanical behavior on the water retention curve was not considered. Therefore, these models cannot be regarded as rigorously coupled hydromechanical models. Possibly

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the first fully coupled hydromechanical model for unsaturated soils was presented by Vaunat et al. [12]. In this model, the effect of the hydraulic behavior on the stress–strain response was considered only through suction, and the deformation influenced the water retention curve through the linkage of the water ratio to the suction. Later, more coupled hydromechanical models were presented. The degree of saturation and suction were always used to account for the influence of the water retention properties on the mechanical behavior [13–22]. The influence of deformation on the hydraulic behavior, however, was represented by shifting the bounding surface of the water retention curve [13,16,21,22] or by incorporating the void ratio [19,23] and porosity [15].

In these constitutive models, the effect of capillary bonding was not directly considered, and the influence of the mechanical behavior on water retention was usually represented using a phenomenological methodology. Very recently, we developed a mechanical model for unsaturated soils by defining a new bonding factor, the bonding force per unit cross-sectional area, to directly represent the effect of the bonding stress on the mechanical behavior [24]. This factor provided the capability to reproduce many important features of the mechanical behavior of unsaturated soils. Additionally, we developed a hysteretic water retention curve model for deformable soils based on the change in the pore size distribution [25]. This model is able to properly predict the irreversible and/or reversible variation in the degree of saturation induced by suction and/or the void ratio. Because the above two models have not been integrated into a fully coupled model, laboratory data on water retention behavior or on the variation of the void ratio must be used in the mechanical or hydraulic model.

In the last two decades, important progress has been made in the development of coupled hydromechanical models that are compatible with thermodynamic laws. In the work of Houlsby [26] and Zhao et al. [27], unsaturated soil is regarded as a three-phase medium. Gray and Schrefler [28] and Nikooee et al. [29] investigated the effective stress in unsaturated soils in thermodynamic equilibrium with an emphasis on, respectively, the solid–fluid interfacial effects and the bonding effects. Within the thermodynamic framework, Santagiuliana and Schrefler [30] discussed the Helmholtz free energy and the dissipation function. Muraleetharan et al. [31] developed coupling relations between the plastic strains and capillary hysteresis. Li [32,33] focused on the work–energy–dissipation relations in an open multiphase thermodynamic system. Coussy et al. [34] developed the thermodynamics of hardening plasticity for unsaturated soils, in which the free energy is considered. Following Coussy et al. [34], Dangla and Pereira [35] presented a thermodynamic analysis of unsaturated soils that included interfaces in the analysis of the mechanical behavior.

In this study, following the thermodynamic framework of Li [32,33] and Coussy et al. [34], the thermodynamics of coupled mechanical deformation and hydraulic hysteresis in unsaturated soils is introduced, and a general formulation is derived for the energy dissipation associated with the irreversible mechanical deformation and the irreversible hydraulic path. By relating the inter-particle bonding factor in the newly developed mechanical model to the hardening force, the newly developed mechanical model [24] is incorporated into the thermodynamic framework, and a void ratio-dependent hysteric water retention curve model [25] is then incorporated into the formulation of a coupled model. We prove that the dissipation associated with the coupled processes is non-negative whenever plastic strains and/or an irreversible change in the degree of saturation occur, demonstrating that our coupled model satisfies the second law of thermodynamics. We validate the model by comparison with experimental data sets and obtain good agreement. The remainder of this paper is organized as follows. In Section 2, the thermodynamic framework is

introduced, based on the thermodynamics of coupled processes, and the general formulation for the energy dissipation is derived. In Section 3, a coupled hydraulic-mechanical model based on two newly developed models [24,25] is incorporated into the thermodynamic framework. In Section 4, the thermodynamic consistency of the coupled model is demonstrated to prove that the dissipation associated with elastoplastic loading and/or the main drying and wetting processes is strictly positive. Finally, in Section 5, the coupled hydromechanical model is validated by comparison with several experimental data sets to demonstrate the performance of the model.

2. Thermodynamic framework

Within the thermodynamic framework, a thermodynamic potential is first defined to enable the derivation of state equations relating the internal state variables to the stress-like conjugate thermodynamic forces [36–38]. For unsaturated soils of incompressible solid grains in isothermal conditions, the total strain of the soil skeleton, ε, and the degree of water saturation, S_r, can be selected as observable variables. In a dissipative process, ε can be divided into two internal variable components: the reversible (elastic) part ε^e and the irreversible (plastic) part ε^p, such that

$$\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p \qquad (1)$$

The internal variables are needed to represent the hardening of unsaturated soil during irreversible loading. In this study, the isotropic hardening behavior is denoted by the internal variable ξ_a, and the internal variable associated with the locked energy during an irreversible water retention process is denoted by ξ_b. From the reversible part of the total strain ($\varepsilon^e = \varepsilon - \varepsilon^p$) and the degree of saturation, the Helmholtz free energy can be written in the following generalized form:

$$\psi = \psi(\varepsilon^e, S_r, \xi_a, \xi_b) \qquad (2)$$

As in the decomposition of the total strain, the degree of saturation S_r is also commonly decomposed into a reversible part S_r^e and an irreversible part S_r^p in unsaturated soil mechanics (e.g., [21,30–32,39–41]) to account for the changes occurring in S_r due to reversible hydraulic processes (represented by the scanning curves) and the irreversible changes occurring in S_r due to irreversible hydraulic processes (i.e., the main drying and main wetting curves). Consequently, one obtains

$$\dot{S}_r = \dot{S}_r^e + \dot{S}_r^p \qquad (3)$$

Due to the presence of the soil skeleton, water, air and the solid–water–air interface within unsaturated soils, and based on the assumption that there is no exchange of entropy or internal energy among the three phases, the free energy can be split into three parts [34], including (i) the elastic free energy ψ_s associated with a reversible mechanical process, (ii) the locked energies Z_1 and Z_2 associated with irreversible mechanical and hydraulic processes, respectively, and (iii) the elastic interface energy related to a reversible hydraulic process U.

The elastic free energy ψ_s, which depends on reversible variables, is a function of ε^e and S_r^e. For the locked energy Z_1, Gallipoli et al. [42] and Sun et al. [43] found that the degree of saturation plays an important role in the irreversible (plastic) deformation process, and Z_1 is therefore dependent on S_r and the internal variable ξ_a. In the hydraulic process, the locked energy Z_2 is related to the internal variable ξ_b and the elastic interface energy depends on S_r^e. Based on the above considerations, the Helmholtz free energy ψ can be additively rewritten as

$$\psi = \psi_s(\varepsilon^e, S_r^e) + Z_1(S_r, \xi_a) + Z_2(\xi_b) + U(S_r^e) \qquad (4)$$