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Research Paper

Slope stability analysis using the limit equilibrium method and two finite element methods

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ABSTRACT

In this paper, the factors of safety and critical slip surfaces obtained by the limit equilibrium method (LEM) and two finite element methods (the enhanced limit strength method (ELSM) and strength reduction method (SRM)) are compared. Several representative two-dimensional slope examples are analysed. Using the associated flow rule, the results showed that the two finite element methods were generally in good agreement and that the LEM yielded a slightly lower factor of safety than the two finite element methods did. Moreover, a key condition regarding the stress field is shown to be necessary for ELSM analysis.

1. Introduction

The limit equilibrium method (LEM) is widely used by researchers and engineers conducting slope stability analysis. The most common limit equilibrium techniques are methods of slices, such as the ordinary method of slices (Fellenius) and the Bishop simplified, Spencer, and Morgenstern-Price methods. The slices technique is well known to be a statically indeterminate problem and is solved by assuming a distribution of internal forces. Consequently, the results obtained from particular methods can vary based on the different assumptions used.

Slope stability analysis using the finite element method has been widely accepted in the literature for many years. The SRM and ELSM are the main finite element slope stability methods currently employed. Comparisons between the LEM and finite element analyses of slope stability illustrate the advantages and limitations of these methods for practical engineering problems.

The SRM was used for slope stability analysis as early as 1975 by Zienkiewicz et al. [1]. This method was later termed the "shear strength reduction technique" by Matusi and San [2], but the name is now typically shortened to the "strength reduction method". During the last ten years, many researchers have applied the SRM to analyse slope stability problems and have compared the SRM and LEM [3–5]. The primary advantage of the SRM is that the critical slip surface is found automatically from the shear strain, which increases as the shear strength decreases. However, the SRM suffers from the important limitation [3] of being unable to locate other "slip" surfaces (i.e., local minima).

The enhanced limit slope stability method calculates stresses using the finite element method and searches for the critical slip surface with the minimum FOS. Brown and King [6] applied this method to analyse the slope stability with a linear elastic soil model. Later, the method was named the "enhanced limit" slope stability method by Nalyor [7]. The definitions of different FOSs in this method have been summarised, and the formulation by Kulhawy was termed the "enhanced limit strength" method (ELSM) by Fredlund [8]. The primary task of the ELSM is to locate the critical slip surface using mathematical optimisation. Many methods have been proposed to identify the critical slip surface based on different optimisation methods, such as the dynamic programming method [9–11], the Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm [12], pattern search [13], and particle swarm optimisation [14].

Researchers have compared the results from the SRM and LEM and those from the ELSM and LEM. However, there are few com-

Abbreviations: LEM, limit equilibrium method; SRM, strength reduction method; ELSM, enhanced limit strength method; FOS, factor of safety; SRF, strength reduction factor; PSO, particle swarm optimisation.

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parisons of the results among all three methods. Furthermore, extremely few studies compare the critical slip surfaces found by these methods; instead, the FOS is the primary parameter of interest. In this paper, the locations and shapes of the slip surfaces and the factors of safety of the slope stability calculated using the LEM, SRM and ELSM based on the assumption that the soil satisfies the Mohr–Coulomb failure criterion are compared. The elastic-perfectly plastic Mohr–Coulomb model is used in both finite element methods. Using the same shear strength parameters, closer agreement can be expected between the finite element method predictions and LEM results. Moreover, the elastic-perfectly plastic model can predict the behaviour of actual soil better than the rigid plastic model used by the LEM. To locate critical slip surfaces in the ELSM, a search technique combining particle swarm optimisation with pattern search is proposed.

2. Comparisons between the definition of the factors of safety

In slope stability analysis, the FOS describes the structural capacity of an embankment or slope, either natural or excavated, beyond the expected or actual loads. In this work, comparisons are performed to determine the correlation among the factors of safety of these three methods.

2.1. LEM

The LEM defines the factor of safety (FOS) as follows:

$$\mathrm{FOS} = \frac{\text{shear strength of soil}}{\text{shear stress required for equilibrium}} \tag{1}$$

Eq. (1) can also be expressed as follows:

$$\tau_i = \frac{\tau_{f_i}}{\mathrm{FOS}} = \frac{c' + \sigma_i' \tan\varphi'}{\mathrm{FOS}} = \frac{c'}{\mathrm{FOS}} + \sigma_i' \frac{\tan\varphi'}{\mathrm{FOS}} \tag{2}$$

where τ_i and σ_i' are the shear stress and effective normal stress at the ith slice of the slip surface, respectively, and c' and φ' are the cohesion and internal friction angle, respectively. In other words, the FOS is "the factor by which the shear strength of the soil would have to be divided to bring the slope into a state of barely stable equilibrium" [15]. The critical slip surface is the surface corresponding to the minimum value of the FOS; this minimum value is the "true" factor of safety.

2.2. SRM

In the SRM, the FOS is defined as the factor by which the original shear strength parameters must be divided to bring the slope to the point of failure [4]. The factored shear strength parameters c_f' and φ_f' are given by

$$c_f' = \frac{c'}{\mathrm{SRF}} \tag{3}$$

$$\varphi_f' = \arctan\left(\frac{\tan\varphi'}{\mathrm{SRF}}\right) \tag{4}$$

where SRF is a strength reduction factor. The FOS is equal to the value of the SRF that causes the slope to fail. Griffiths and Lane noted that this definition of the FOS is exactly the same as that used in the LEM [4].

2.3. ELSM

In the ELSM, for an arbitrary slip surface L, the FOS can be defined as

$$\mathrm{FOS} = \frac{\sum_{i=1}^{n} \tau_{f_i} \Delta L_i}{\sum_{i=1}^{n} \tau_i \Delta L_i} = \frac{\int \tau_f dL}{\int \tau dL} \tag{5}$$

where n is the number of discrete segments along L and ΔL_i is the length of segment i. Based on the stresses calculated by the finite element method, the shear stress τ_i and the effective normal stress σ_i' can be expressed in the form of Eq. (2):

$$\tau_i = \frac{\tau_{f_i}}{\mathrm{FOS}(\Delta L_i)} = \frac{c' + \sigma_i' \tan\varphi'}{\mathrm{FOS}(\Delta L_i)} = \frac{c'}{\mathrm{FOS}(\Delta L_i)} + \sigma_i' \frac{\tan\varphi'}{\mathrm{FOS}(\Delta L_i)} \tag{6}$$

where $\mathrm{FOS}(\Delta L_i)$ is the function of the strength reduction factor for segment i.

Substituting Eq. (6) into (5),

$$\frac{\sum_{i=1}^{n} \tau_{f_i} \Delta L_i}{\sum_{i=1}^{n} \tau_i \Delta L_i} = \frac{\sum_{i=1}^{n} (c' + \sigma_i' \tan\varphi') \Delta L_i}{\sum_{i=1}^{n} \left(\frac{c'}{\mathrm{FOS}(\Delta L_i)} + \sigma_i' \frac{\tan\varphi'}{\mathrm{FOS}(\Delta L_i)} \right) \Delta L_i} = \frac{\int \tau_f dL}{\int \frac{\tau_f}{\mathrm{FOS}(\Delta L_i)} dL} \tag{7}$$

Based on the first mean value theorem for integration, the lower right side of Eq. (7) can be transformed into

$$\int \frac{\tau_f}{\mathrm{FOS}(\Delta L_i)} dL = \frac{\int \tau_f dL}{\mathrm{FOS}} \tag{8}$$

According to Eqs. (7) and (8), the FOS in the ELSM can be expressed in the same form as Eq. (5).

For a slip line that is neither straight nor circular, the physical meaning of the FOS in the ELSM has been questioned by several researchers because the integration in Eq. (5) is neither the summation of force vectors in space nor the summation of the projections of force vectors in a fixed direction [16]. However, several researchers have already proven that the definition can be considered acceptable in practical application [8,9,12,13].

The common definitions of the LEM and SRM can be interpreted as corresponding to two different methods used to obtain a set of reduction strength parameters that cause the slope to reach its critical limit equilibrium state. Based on the derivations in the ELSM, it can be shown that the definition of the FOS is also established based on the strength reduction, as in the LEM and SRM. Furthermore, the FOS in the ELSM is the "average" strength reduction factor along the slip surface based on the first mean value theorem for integration

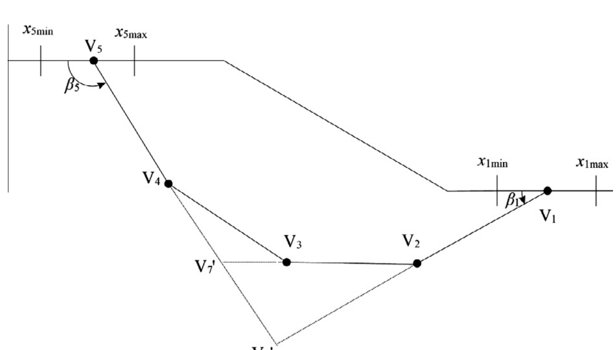

Fig. 1. Generation of the trial slip surface.

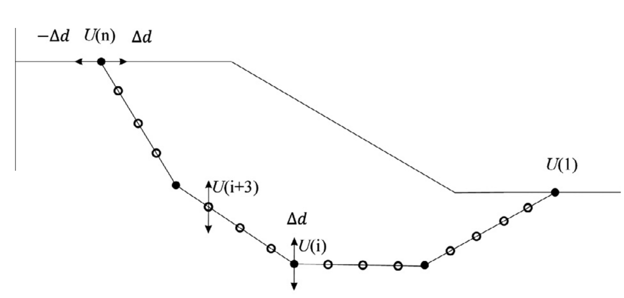

Fig. 2. Discretisation of the slip surface.