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Research Paper

Consolidation around a tunnel in a general poroelastic medium under anisotropic initial stress conditions

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ABSTRACT

In this study, a solution for the response of a poroelastic medium around a deeply excavated circular tunnel is analytically formulated based on Biot's consolidation theory. The proposed solution considers the initial anisotropic stress in the medium around lined or unlined tunnels and is an improvement over previous solutions (Carter and Booker, 1982, 1984) because it considers Biot's coefficient, the combined compressibility of liquid and solid phases in the porous medium, and a thick-walled liner. The solution is expressed using the Laplace transform domain, and numerical inversion techniques are used to obtain real time domain results. The new solution is verified by comparing it with previous analytical solutions and numerical results obtained using the commercial finite element software COMSOL Multiphysics (COMSOL AB, Stockholm, Sweden). Parametric studies are performed to determine the influences of Biot's coefficient and the combined compressibility and liner properties during the consolidation process.

1. Introduction

During the past several decades, disposal in deep geological formations has become a promising option in many countries for managing high-level radioactive waste. Feasibility studies for radioactive waste repositories sometimes require the construction of access and experimental tunnels that reach several hundred metres below the ground for in-situ research. Proper stress, displacement and pore pressure variation estimates during and after tunnel excavation are important when designing such repositories.

Although numerical methods have been extensively applied to solve tunnel problems, analytical approaches remain useful and may be advantageous because they can provide direct qualitative insights and verify numerical tools. Many analytical solutions have been developed for circular tunnels that have been excavated in isotropic and homogeneous media. Previously, the media surrounding tunnel liners was considered to behave as a single-phase material; thus, the solutions were only relevant to short- (or undrained) and long-term (or drained) conditions [17,19]. Carter and Booker [3], Carter and Booker [4] improved the solution by considering a poroelastic saturated medium and derived the time-dependent consolidation for unlined tunnels and tunnels supported by thin-walled liners. In their solutions, the solid grains and the liquid phase were treated as incompressible, which is a widely accepted assumption for soft and fully saturated soils but may not be suitable for rock substrates or nearly saturated soils.

In this study, a hydro-mechanical (HM) coupled analysis based on Biot's consolidation theory [1] is presented for a deeply excavated circular tunnel in an isotropic, poroelastic medium of infinite extent. Within the framework of Biot's theory, the compressibility of solid particles and the liquid phase is addressed in terms of Biot's coefficient, ξ, and the combined compressibility, β. Initially anisotropic stresses are included in the solution because they often occur in situ and play very important roles. Moreover, the thin-walled liner governed by cylindrical shell theory in the solution proposed by Carter and Booker [4] is extended to include a more general, thick-walled, permeable or impermeable liner.

Rice and Cleary [20] recast Biot's work using contemporary terminology and showed that the governing equations for the general case of arbitrarily compressible constituents are not significantly more complicated. However, the effective stress coefficient and poroelastic parameters must be modified. The relevance of Biot's and Terzaghi's effective stress for failure phenomena around a deep borehole was discussed by Cornet and Fairhurst [9]. In addition, Detournay and Cheng [11] comprehensively analysed the poroelastic effects around deep tunnels and boreholes. This analysis has been extended to elasto-plastic media [5], a heated poroelastic medium [16], large strain elasto-plastic soil [25], cross-anisotropic rock [14,15], and other materials.

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Notation

ξ	Biot's coefficient of porous medium
β	combined compressibility of porous medium
K_b	bulk modulus of the porous medium
K_s	bulk modulus of the solid grains of the porous medium
G	shear modulus of the porous medium
v	drained Poisson's ratio
σ	total normal stress
σ'	effective normal stress
p	pore liquid pressure
K_l	bulk modulus of the liquid phase
n	porosity of the porous medium
M	sometimes called Biot's modulus an accounts for the compressibilities of solid and liquid constituents
a	radius of the lining intrados
b	radius of the lining extrados
r	radius
θ	rotation angle
S_l	pore liquid saturation degree
σ_V	total vertical stress
σ_H	total horizontal stress
σ_m	total mean stress
σ_d	deviatoric stress
σ_r, σ_θ, $\tau_{r\theta}$	total radial, total circumferential and shear stresses
σ_{r0}, $\sigma_{\theta0}$, $\tau_{r\theta0}$, p_0	initial stresses and pore pressure
u, v	radial and circumferential displacements
ε_v, ε_r, ε_θ, $\gamma_{r\theta}$	volumetric, radial, circumferential and shear strains
λ	Lame parameter
k	Darcy's hydraulic conductivity of the porous medium
γ_l	unit weight of the liquid phase
B	Skempton's pore pressure coefficient
v_u	undrained Poisson's ratio
C	consolidation or diffusivity coefficient
G_l	shear modulus of liner
v_l	Poisson's ratio of liner
E_l	Young's modulus of liner
E_l^*	"effective" modulus of the liner $E_l^* = E_l/(1-v_l^2)$
$U(r, t)$, $V(r, t)$	Fourier coefficients of radial and circumferential displacement
$P(r, t)$	Fourier coefficients of pore water pressure
$S_r(r, t)$, $S_\theta(r, t)$	Fourier coefficients of radial and circumferential stress
$T_{r\theta}(r, t)$	Fourier coefficients of shear stress
$E_v(r, t)$, $\Omega_r(r, t)$	Fourier coefficients of volumetric and deviatoric strain

1.1. Biot's coefficient

Classical quasi-static Biot's theory [1] describes coupled elastic deformations and diffusive flow in fully connected porous media Biot's coefficient is defined as the ratio of the fluid volume gained (or lost) in a material element to the volume change of the porous medium and is expressed conventionally as follows [2,18]:

$$\xi = 1 - \frac{K_b}{K_s}, \tag{1}$$

where ξ is Biot's coefficient; K_s is the bulk modulus of the solid grain; and K_b is the drained bulk modulus of the porous medium related to the stiffness of the porous medium by $K_b = \frac{2G(1+v)}{3(1-2v)}$. Here, G and v are the drained shear modulus and Poisson's ratio, respectively. For soft soil ($K_b \ll K_s$), the compressibility of the solid phase is negligible relative to that of the drained bulk material and $\xi \approx 1$.

Biot's coefficient can be used to calculate the effective stress as follows:

$$\sigma' = \sigma + \xi p, \tag{2}$$

where σ and σ' are the total stress and effective stress, respectively, with positive tension, and p is the pore liquid pressure with positive compression. Because Biot's coefficient always satisfies $\xi \leqslant 1$, this equation implies that the pore pressure is not totally effective for counteracting the effects of confining pressure for changing the bulk volume. Consequently, Biot's coefficient is also known as an effective stress coefficient. Table 1 lists several poroelastic constants for different rocks.

1.2. Combined compressibility of the pore fluid and solid phase

The combined compressibility of the pore fluid and solid phase, β, also known as the storage coefficient, is defined as the volume change of the pore fluid per unit volume of porous medium as a result of a unit of increasing pore pressure. For an ideal porous material characterised by a fully connected pore space (porosity n) and a microscopically homogeneous and isotropic matrix, the combined compressibility can be written as follows:

$$\beta = \frac{1}{M} = \frac{n}{K_l} + \frac{\xi - n}{K_s}, \tag{3}$$

where the constant M is sometimes called Biot's modulus, and K_l is the bulk modulus of the liquid phase ($K_{l0} = 2.2$ GPa for gas-free water at 25 °C). For a highly compressible fluid ($K_l \ll K_s$), the approximated expression for β is $\beta = n/K_l$.

It is important to consider the compressibility of the pore liquid during consolidation in the following two types of porous media [6].

(i) *Saturated porous rock:* For water-saturated rock, the pore water is not effectively incompressible. In many cases, the modulus of the porous rock (K) and the modulus of the air-free liquid (K_{l0}) are similar in magnitude. Therefore, the compressibility of the pore fluid should be considered [21].

(ii) *Nearly saturated soil:* When the degree of soil saturation is nearly 100%, the liquid phase becomes continuous, and the gas phase becomes discontinuous and occluded in the liquid phase in the form of bubbles. The pore water containing bubbles behaves as a "homogeneous compressible fluid". A simple analysis by Verruijt [22] indicates the following upper boundary for the compressibility of the pore fluid mixture:

$$\frac{1}{K_l} = \frac{1}{K_{l0}} + \frac{1 - S_l}{P_{l0}}, \quad 1 - S_l \ll 1, \tag{4}$$

where S_l is the degree of pore liquid saturation, and P_{l0} is the absolute pore liquid pressure. This condition prevails at a degree of saturation with $1 - S_r \ll 1$. Eq. (4) indicates that even very small amounts of gas in soils will dramatically reduce the bulk modulus of the pore fluid.

1.3. Thick-walled liner

In an earlier investigation conducted by Carter and Booker [4], the liner is assumed as a thin elastic tube in intimate contact with