Computers and Geotechnics 66 (2015) 96–112

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

A robust finite volume model to simulate granular flows

S. Yavari-Ramshe [a,*], B. Ataie-Ashtiani [a], B.F. Sanders [b]

[a] Department of Civil Engineering, Sharif University of Technology, PO Box 11155-9313, Tehran, Iran
[b] The Henry Samueli School of Engineering, Department of Civil and Environmental Engineering, University of California, Irvine, USA

ARTICLE INFO

Article history:
Received 21 June 2014
Received in revised form 10 November 2014
Accepted 22 January 2015
Available online 11 February 2015

Keywords:
Finite volume method
Mudslide
Granular flow
Landslide
Coulomb friction law

ABSTRACT

This paper introduces a well-balanced second-order finite volume scheme, based on the *Q*-scheme of Roe, for simulating granular type flows. The proposed method is applied to solve the incompressible Euler equations under Savage–Hutter assumptions. The model is derived in a local coordinate system along a non-erodible bed to take its curvature into account. Moreover, simultaneous appearance of flowing/static regions is simulated by considering a basal friction resistance which keeps the granular flow from moving when the angle of granular flow is less than the angle of repose. The proposed scheme preserves stationary solutions up to second order and deals with different situations of wet/dry transitions by a modified nonlinear wet/dry treatment. Numerical results indicate the improved properties and robustness of the proposed finite volume structure. In addition, the granular flow properties are estimated with a computational error of less than 5%. These errors are consistently less than those obtained by using similar existing finite volume schemes without the proposed modifications, which can result in up to 30% overestimation.

1. Introduction

Natural granular flows like landslides, mudslides, snow avalanches and rockslides are natural hazards that may impose fatalities and significant economical damages. These flows are associated with soil erosion and sedimentation into rivers and valleys [1,18,33], seabed topography change, and soil or surface/ground water contamination [64]. Moreover, on the shores of a water body, they may be followed by resulting impulsive waves and their subsequent dam overtopping [6,7,9–11,14,63,82] or run-up to coastal areas [36,80] as a secondary hazard. In order to conduct hazard analysis and protect settled areas, predictions of the flow thickness and velocity of the slide are needed [58,62,72]. To this end, a number of numerical studies have been performed based on different numerical approaches.

Savage and Hutter [70] pioneered the study of rock, snow and ice avalanches based on shallow water equations under hydrostatic assumption, using two finite difference methods, one of Lagrangian and the other of Eulerian. Their theory was verified to be in an excellent agreement with laboratory experiments [39,46,52,70]. Many of the available numerical models apply the Savage–Hutter (SH) type considerations to describe the behavior of granular type flows [30,44,45,58,65,75,81]. This fact also confirms the ability and efficiency of these assumptions in recitation of the granular flow behavior [51]. SH type models are based on the shallow water equations considering a Coulomb friction term as the flow/bottom interaction [70]. The constitutive relation of the granular material is also defined based on the Mohr–Coulomb criteria; i.e. the normal stresses are related to the longitudinal stresses by a factor *K* (the earth pressure coefficient) [70]. In 1991, the SH formulation was transferred to a local coordinate system for considering the bed curvature effects [71]. Gray et al. [38] extended this model to two dimensions. Wieland et al. [81] used a mixed FVM–FDM (Finite Volume Method–Finite Difference Method) to discretize the two dimensional SH model. The effects of the bed erosion were inserted in this model by Pitman et al. [65] who applied a Godunov type FVM to discretize the model equations. Denlinger and Iverson [31] extended the three dimensional version of a SH type model using Harten, Lax and Van Leer contact (HLLC) finite volume scheme. More studies have been performed on behavior of granular type flows based on different rheologies and governing equations using FDM [2,42,49,62,75], FVM [23,32,53,58,61,83], FEM (Finite Element Method) [4,27,28,35], SPH (Smoothed Particle Hydrodynamics) [59], or a combination of these schemes [3,81].

A comprehensive review of these studies is summarized in Table 1. This table shows the previous numerical models including their governing equations, considered rheology, numerical

* Corresponding author. Tel.: +1 (714) 861 9220; fax: +98 (21) 6601 4828.
E-mail addresses: yavari@mehr.sharif.edu (S. Yavari-Ramshe), ataie@sharif.edu (B. Ataie-Ashtiani), bsanders@uci.edu (B.F. Sanders).

http://dx.doi.org/10.1016/j.compgeo.2015.01.015
0266-352X/

Nomenclature

A	coefficient matrix
b	bottom level
c	characteristic wave velocity
D	diagonal matrix of eigenvalues
df	generalized Roe flux difference
Err	computational error
F	numerical flux matrix
G	source term matrix
G_1	source term matrix concerning bed level
G_2	source term matrix concerning bed curvature
G_3	first θ related part of the flux term
G_4	second θ related part of the flux term
$\vec{g}$	gravitational acceleration vector
g	gravitational acceleration
H	granular flow depth vertical to the bed
H'	characteristic depth
h	granular flow depth $(h')/\cos^2\theta$
h'	granular flow depth
I	computational cell
Id	identity matrix
J	Jacobean of transformation matrix
K	earth pressure coefficient
κ	eigenvector
L	characteristic length
m	number of computational grids
n	number of time steps
n_s	unit normal vector of flow surface
n_b	unit normal vector of bottom
P	pressure tensor
P_{XX}	normal pressure along X
P_{ZZ}	normal pressure along Z
P_{ZX}	longitudinal stress along X
P_{XZ}	longitudinal stress along Z
P_{xx}	normal pressure along x
P_{zz}	normal pressure along z
P_{zx}	longitudinal stress along x
P_{xz}	longitudinal stress along z
P_1	$\kappa D \kappa^{-1}$
$P^{\pm}$	projection matrixes $1/2\kappa(Id \pm \text{sgn}(D))\kappa^{-1}$
Q	matrix characteristic of a Q scheme
$\underset{\smile}{q}$	flow discharge hu
q	depth-averaged flow discharge $h\,\underset{\smile}{u}$
q^*	predicted flow discharge in the first step
r	$\Delta t/\Delta x$
S	numerical source term matrix
S_1	numerical source term matrix related to bed level
S_2	numerical source term matrix related to bed curvature
S_3	1st numerical θ related part of the flux term matrix
S_4	2nd numerical θ related part of the flux term matrix
T	Coulomb friction matrix
T^*	Coulomb friction matrix of the corrector step
t	time
U	flow velocity parallel to the bottom
$\underset{\smile}{U}_b$	sliding velocity along bottom
U	depth-averaged velocity parallel to bottom
u	horizontal flow velocity
$\bar{u}$	Roe-averaged velocity
V	flow velocity perpendicular to the bottom
V'	flow velocity vector (u, v)
v	vertical flow velocity
W	unknown matrix $[h\,q]$
W^*	predicted values in the first step $[h\,q^*]$
W^+	exact solution of nonlinear Riemann problem in the right edge of wet/dry transition intercell
W^-	exact solution of nonlinear Riemann problem in the left edge of wet/dry transition intercell
X	local coordinate component along non-erodible bed
$\vec{X}$	cartesian coordinate vector (x,z)
$\vec{X}'$	local coordinate vector (X,Z)
x	horizontal component of Cartesian coordinate system
Y_1	a state value
Y_2	a state value
Z	local coordinate component perpendicular to the bed
z	vertical component of Cartesian coordinate system
ρ	density of granular mass
θ	local slope angle of the bed
δ	basal friction angle
δ_0	angle of repose
ϕ	internal friction angle of granular material
ϕ'	a numerical flux function
ε	small parameter of dimensional analysis
$\Im$	Coulomb friction term
σ_c	critical friction resistance of bottom
λ	eigenvalue
Δx	computational cell size
Δt	computational time step
τ_{crit}	critical longitudinal stress of the bottom
∇	gradient vector $(\partial/\partial x,\ \partial/\partial z)$
γ	a small parameter $\in (0, 1)$

approaches and numerical schemes. Based on this review, FVM and FEM have been more popular than FDM because of using the integral form of conservation laws which is closer to the physics [55,73]. FVM has also the advantage of preserving conservation of mass and momentum in multidimensional physical systems like granular avalanches where rapid transitions between flowing and static states are common [55]. The new approach of SPH, which has been lately used by many researchers, e.g. [5,8,12,13,59], is not efficient in simulating the situations where flow encounters unexpected corners or constrictions [30].

The SH type formulations are applied in the present model to describe the behavior of the granular flow. The present SH type model has two special properties. It takes bed curvature effects and flow dynamic/static regions into account. Based on the previous studies, bottom curvatures have noticeable effects on the behavior of granular type flows [20,30,34,42,67]. Lately, two new SH models have been introduced by Bouchut et al. [20] over a general bottom. The first model considers small variations of the bed curvature and the second one is dealing with general bottom topographies. The present SH type model applied the first hypothesis, i.e. a small variation of the curvature. Accordingly, the model equations are derived in a local coordinate system along with the bed to take its curvature into account. This model differs from original SH model through a new curvature term which is required to obtain the energy inequality and to satisfy the stationary solutions regarding water at rest [20]. Moreover, in the present model, a critical stress is defined to stop the granular layer from moving when its angle is less than the angle of repose [19,34]. This second property is especially important when the flow is supposed to be shallow which results in simultaneous existence of the flowing and the static regions [72].

Effective and robust numerical solution of the system of model equations described above is the main focus of this paper. A well-balanced finite volume scheme is proposed which minimize the