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Research Paper

A stabilized single-point finite element formulation for three-dimensional dynamic analysis of saturated soils

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ABSTRACT

A low-order single-point quadrature finite element suitable of dynamic 3D analysis of saturated soils is presented. The new element uses a **u**-*p* formulation to consider the interaction of the pore fluid and solid skeleton. An hourglass stabilization scheme facilitates single-point integration for the solid phase terms, and non-residual pressure field stabilization is used to facilitate equal-order interpolation for the two phases and improve element behavior in the incompressible–impermeable limit. Several numerical examples are presented to verify the new element formulation and demonstrate its response in a geotechnical application.

1. Introduction

Three-dimensional numerical analysis techniques represent an valuable tool for increasing the understanding of the behavior complex geotechnical systems. As noted by Elgamal et al. [1], the advances in software and hardware and developments in numerical simulation tools, such as constitutive models, that have taken place over the last 10–15 years have resulted in increased viability for 3D simulations of saturated soils and other geotechnical problems. Three-dimensional analysis is particularly attractive for the study of problems that possess inherent three-dimensionality, such as the analysis of piles in liquefied and laterally spreading soils [e.g., 2–4], the assessment of the effects of simultaneous horizontal accelerations in two directions, the analysis of differential settlements due to liquefaction [5], and the study of 3D slopes subject to seismic excitation [6]. Though the viability of such studies has increased, these analyses remain computationally intensive, and any increased efficiency that can be gained is beneficial.

The current paper proposes a new low-order eight-node hexahedral element with single-point quadrature and a **u**-*p* formulation for the consideration of saturated soils subject to dynamic loadings. Hourglass stabilized reduced integration [7–10] is used to control the spurious deformation modes associated with under-integration, and an enhanced assumed strain field [11,12] is incorporated into the stabilization terms to address volumetric and shear locking. The stabilization terms for the solid phase are evaluated using a unique analytical pre-integration technique to eliminate the need for numerical integration and further increase the efficiency of the formulation. Linear interpolation functions are used for the displacement and pressure fields, therefore, the element does not inherently satisfy the *inf-sup* condition [13,14], and a non-residual based stabilization scheme, modeled after the work of Huang et al. [15,16], is implemented to improve element performance in the limiting case of an incompressible pore fluid and an impermeable mixture. The general concepts used to develop this new element correspond with those introduced for an efficient plane strain **u**-*p* element formulation previously developed by the authors [17]. The implementation of these schemes for a 3D element results in certain complexities not encountered in plane strain, and these differences are highlighted throughout the discussion.

The proposed element formulation, H1-P1ssp, results in a stable, accurate, and computationally efficient element suitable for three-dimensional dynamic analysis of saturated soils. The solid and fluid phase formulations for this new element are discussed in separate sections following a brief introduction to the general mixed element basis. Details regarding the pre-integration approach for evaluation of the solid phase stabilization terms are provided in the appendices that follow the primary discussion. Several numerical examples are provided to verify the implementation of the element, compare the accuracy of the H1-P1ssp element to corresponding results for higher-order elements, and to demonstrate its use in a complex geotechnical application. Though current computational resources have increased the viabil-

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ity of higher-order elements in 3D analyses, the new element formulation presented here provides a computationally efficient alternative to the common H2-P1 (20 or 27 node) element for most practical purposes. Given a limited level of computational resources, the H1-P1ssp element can enable the analysis of larger and/or more refined problems, can reduce execution times for simpler validation and proof-of-concept analyses, and can enable the completion of larger parameter studies than a corresponding higher-order formulation.

2. General mixed element formulation

Elements developed for modeling saturated porous media, e.g., [18–21], are typically derived from mixed formulations that consider the coupled response of distinct fluid and solid phases, often within the framework of the early work of Biot [22–24]. The H1-P1ssp element uses the **u**-*p* formulation [21] to create such a coupled system, as this approach is particularly attractive in the context of an efficient solution and the assumptions associated with its use are valid in most soil dynamics problems.

The **u**-*p* element formulation describes the coupled system in terms of two variables, the displacement of the solid phase, **u**, and the pressure in the pore fluid, *p*, and is derived from two coupled equations, the equation of motion for the mixture neglecting the acceleration of the fluid,

$$\nabla \cdot (\boldsymbol{\sigma}' - p\mathbf{1}) + \rho\mathbf{g} - \rho\ddot{\mathbf{u}} = \mathbf{0} \tag{1}$$

and the combined equation of motion for the fluid phase and mass balance for the mixture,

$$\mathrm{tr}\dot{\boldsymbol{\varepsilon}} + \frac{n}{K_f}\dot{p} + \nabla \cdot \left[\mathbf{k}\left(-\nabla p + \rho_f \mathbf{g}\right)\right] = 0 \tag{2}$$

where **u** is the displacement of the solid phase, $\boldsymbol{\sigma}'$ is the effective stress, **1** is the second-order identity tensor, ρ is the mixture mass density, $\mathrm{tr}\,\dot{\boldsymbol{\varepsilon}}$ is the volumetric strain rate in the solid phase, n is the porosity, K_f and ρ_f are the pore fluid bulk modulus and mass density, **k** is the permeability tensor, and **g** is a gravitational acceleration vector such that $\rho\mathbf{g}$ and $\rho_f\mathbf{g}$ are a body force vectors for the solid and fluid phases, respectively. In this and all subsequent discussion, compression is taken as negative.

Discretized expressions for this coupled system are obtained via the standard Galerkin technique using the approximations $\mathbf{u} \approx \mathbf{N}_u\mathbf{d}$ and $p \approx \mathbf{N}_p\mathbf{p}$, where $\mathbf{N}_u$ and $\mathbf{N}_p$ are arrays of interpolation functions for displacement and pressure, respectively, and **d** and **p** are vectors of nodal displacements and pore pressures. The resulting expressions are as follows:

$$\mathbf{M}\ddot{\mathbf{d}} + \int_\Omega \mathbf{B}^T\boldsymbol{\sigma}'\,d\Omega - \mathbf{Q}\mathbf{p} = \mathbf{f}_u^{\mathrm{ext}} \tag{3}$$

$$\mathbf{Q}^T\dot{\mathbf{d}} + \mathbf{S}\dot{\mathbf{p}} + \mathbf{H}\mathbf{p} = \mathbf{f}_p^{\mathrm{ext}} \tag{4}$$

where

$$\mathbf{M} = \int_\Omega \mathbf{N}_u^T\rho\mathbf{N}_u\,d\Omega \tag{5}$$

$$\mathbf{Q} = \int_\Omega \mathbf{B}^T\mathbf{1}\mathbf{N}_p\,d\Omega \tag{6}$$

$$\mathbf{S} = \int_\Omega \mathbf{N}_p^T\frac{n}{K_f}\mathbf{N}_p\,d\Omega \tag{7}$$

$$\mathbf{H} = \int_\Omega \nabla\mathbf{N}_p^T\mathbf{k}\nabla\mathbf{N}_p\,d\Omega \tag{8}$$

$$\mathbf{f}_u^{\mathrm{ext}} = \int_\Omega \mathbf{N}_u^T\rho\mathbf{g}\,d\Omega + \int_\Gamma \mathbf{N}_u^T\mathbf{t}\,d\Gamma \tag{9}$$

$$\mathbf{f}_p^{\mathrm{ext}} = \int_\Gamma \mathbf{N}_p^T\mathbf{k}\mathbf{q}\,d\Gamma - \int_\Omega \nabla\mathbf{N}_p^T\rho_f\mathbf{k}\mathbf{g}\,d\Omega \tag{10}$$

where **B** is the standard kinematic matrix for the solid phase, **t** is a surface traction for the solid phase, **q** is a surface flux for the fluid phase, and $\mathbf{1} = [1,1,1,0,0,0]^T$. Definitions for the remaining terms in Eqs. (3)–(10) are provided in subsequent sections that discuss the evaluation of the various solid phase and fluid phase constituents for the H1-P1ssp element. Voight notation is adopted in Eqs. (3)–(10) and in all subsequent development.

3. Formulation for the solid phase

The framework used to evaluate the solid phase terms of Eqs. (3) and (4) is similar to that of the Q1-P1ssp element [17], however, the differences between the 3D and plane strain formulations are significant enough to warrant a full discussion here, even at the expense of some repetition. The primary differences between the two formulations are noted at applicable points in the following discussion.

3.1. Element kinematics

The solid phase kinematic equations for the H1-P1ssp element are developed from the displacement field of Belytschko and Bachrach [25] in order to facilitate a split into two portions: (1) a non-constant (stabilizing) portion associated with the hourglass modes for the element solid phase and (2) a constant portion associated with the remaining deformation modes. The chosen displacement field has the form

$$\mathbf{u} = (a_{0i} + a_{1i}x + a_{2i}y + a_{3i}z + c_{1i}h_{\xi\eta} + c_{2i}h_{\zeta\eta} + c_{3i}h_{\zeta\xi} + c_{4i}h_{\xi\eta\zeta})\mathbf{e}_i \tag{11}$$

where a_{1i}, a_{2i}, a_{3i}, c_{1i}, c_{2i}, c_{3i}, and c_{4i} are scalar coefficients, $\mathbf{e}_i$ are unit vectors, x, y, and z are global coordinates, and

$$h_{\alpha\beta} = \alpha\beta, \qquad \alpha,\ \beta = \xi,\ \eta,\ \zeta \tag{12}$$

$$h_{\xi\eta\zeta} = \xi\eta\zeta \tag{13}$$

are the local coordinate products. Using this form, the nodal displacements,

$$\mathbf{d}_x = [u_1,u_2,u_3,u_4,u_5,u_6,u_7,u_8]^T \tag{14}$$

$$\mathbf{d}_y = [v_1,v_2,v_3,v_4,v_5,v_6,v_7,v_8]^T \tag{15}$$

$$\mathbf{d}_z = [w_1,w_2,w_3,w_4,w_5,w_6,w_7,w_8]^T \tag{16}$$

can be expressed as

$$\mathbf{d}_i = a_{0i}\mathbf{r} + a_{1i}\mathbf{x} + a_{2i}\mathbf{y} + a_{3i}\mathbf{z} + c_{1i}\mathbf{h}_{\xi\eta} + c_{2i}\mathbf{h}_{\zeta\eta} + c_{3i}\mathbf{h}_{\zeta\xi} + c_{4i}\mathbf{h}_{\xi\eta\zeta} \tag{17}$$

where

$$\mathbf{r} = [1,1,1,1,1,1,1,1]^T \tag{18}$$

$$\mathbf{x} = [x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8]^T \tag{19}$$

$$\mathbf{y} = [y_1,y_2,y_3,y_4,y_5,y_6,y_7,y_8]^T \tag{20}$$

$$\mathbf{z} = [z_1,z_2,z_3,z_4,z_5,z_6,z_7,z_8]^T \tag{21}$$

$$\mathbf{h}_{\xi\eta} = h_{\xi\eta}(\boldsymbol{\xi},\boldsymbol{\eta},\boldsymbol{\zeta}) = [1,-1,1,-1,1,-1,1,-1]^T \tag{22}$$

$$\mathbf{h}_{\zeta\eta} = h_{\zeta\eta}(\boldsymbol{\xi},\boldsymbol{\eta},\boldsymbol{\zeta}) = [1,1,-1,-1,-1,-1,1,1]^T \tag{23}$$

$$\mathbf{h}_{\zeta\xi} = h_{\zeta\xi}(\boldsymbol{\xi},\boldsymbol{\eta},\boldsymbol{\zeta}) = [1,-1,-1,1,-1,1,1,-1]^T \tag{24}$$

$$\mathbf{h}_{\xi\eta\zeta} = h_{\xi\eta\zeta}(\boldsymbol{\xi},\boldsymbol{\eta},\boldsymbol{\zeta}) = [-1,1,-1,1,1,-1,1,-1]^T \tag{25}$$

and

$$\boldsymbol{\xi} = [-1,1,1,-1,-1,1,1,-1]^T \tag{26}$$

$$\boldsymbol{\eta} = [-1,-1,1,1,-1,-1,1,1]^T \tag{27}$$

$$\boldsymbol{\zeta} = [-1,-1,-1,-1,1,1,1,1]^T \tag{28}$$