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Research Paper

Deterministic and probabilistic multi-modal analysis of slope stability

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ABSTRACT

Traditional slope stability analysis involves predicting the location of the critical slip surface for a given slope and computing a safety factor at that location. However, for some slopes with complicated stratigraphy several distinct critical slip surfaces can exist. Furthermore, the global minimum safety factor in some cases can be less important than potential failure zones when rehabilitating or reinforcing a slope. Existing search techniques used in slope stability analysis cannot find all areas of concern, but instead converge exclusively on the critical slip surface. This paper therefore proposes the use of a holistic multi modal optimisation technique which is able to locate and converge to multiple failure modes simultaneously. The search technique has been demonstrated on a number of benchmark examples using both deterministic and probabilistic analysis to find all possible failure mechanisms, and their respective factors of safety and reliability indices. The results from both the deterministic and probabilistic models show that the search technique is effective in locating the known critical slip surface while also establishing the locations of any other distinct critical slip surfaces within the slope. The approach is of particular relevance for investigating the stability of large slopes with complicated stratigraphy, as these slopes are likely to contain multiple failure mechanisms.

1. Introduction

A wide range of methods have been developed to solve slope stability problems. Methods such as the Limit Equilibrium Method (LEM), Strength Reduction Method (SRM) and the Limit Analysis method are commonly used. The LEM and SRM methods are most widely adopted by researchers and practitioners [1]. Both methods, although originally deterministic, can be easily adapted to suit probabilistic models. In a deterministic analysis stability is evaluated in terms of a factor of safety (FOS) which is based on fixed parameter values. In a probabilistic analysis, every variable is assigned a statistical distribution, and stability is evaluated in terms of a reliability index (β) or a probability of failure (P_f).

In most LEM analysis, the critical slip surface which provides the minimum FOS or β is unknown and needs to be determined using either a trial and error approach or optimisation techniques [2]. As the processing power of personal computers has increased, Monte Carlo simulations have been adopted into commercial software packages such as SLOPE/W and SOIL VISION. The SRM method has been adopted into several well-known finite element (PLAXIS, GEO5) or finite difference (FLAC) programs. To carry out a probabilistic analysis of stability using SRM, a spatially correlated soil field is typically developed using random field theory and then solved using the finite element or finite difference methods [3].

Due to the complexity of the problem, both LEM and SRM have their own advantages and disadvantages. LEM requires less detailed knowledge about the site and in the majority of cases provides satisfactory results when pore water pressure is correctly modelled. Because LEM is currently the most widely used method of evaluating slope stability in practice, extensive research has been undertaken in an effort to improve its performance. Particular emphasis has been placed on finding the global critical slip surface in order to obtain the minimum FOS and β and the associated maximum probability of failure (P_f) of a slope. Comparatively, little research has been completed on slopes which could develop a number of critical slip surfaces with similar minimum FOS and β (or maximum P_f) values. There are cases where the global minimum is of little practical importance, e.g. when the critical slip surface is too shallow to have any severe consequences, or when a slope is susceptible to multiple failure mechanisms,

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e.g. slopes with multiple benches and/or layers. Determination of "critical" slip surfaces is affected by the experience of the engineer or researcher, as only one failure mechanism can be identified in each trial. As noted by Griffiths et al. [4] for slopes with multiple failure mechanisms, failure to detect some of the failure surfaces could lead to unsafe design, particularly for cases where remedial measures such as soil reinforcement are required. Therefore, a robust algorithm is required to determine all potential critical slip surfaces considering multiple failure mechanisms.

This paper uses a multi-modal particle swarm optimisation technique to analyse the stability of slopes with multiple failure mechanisms. Both deterministic and probabilistic analyses were carried out, using an LEM based model to determine the FOS and β. Several benchmark problems were analysed to demonstrate the effectiveness of the method. The results showed that the proposed method could efficiently determine multiple 'critical' slip surfaces for each failure mode and solve for the related FOS and β simultaneously.

2. Deterministic and probabilistic analysis of slopes

The factor of safety of a slope can be defined in LEM as the ratio between resistance and disturbance along a potential slip surface:

$$\mathrm{FOS} = \frac{\text{resistance}}{\text{disturbance}} \tag{1}$$

There are many published methods available which can be used to obtain the resistance and disturbance, most of which are based on the method of slices. For the slope shown in Fig. 1, using the simplified Bishop's method of slices [5], the factor of safety of a slope can be defined as:

$$\mathrm{FOS} = \frac{\sum_{i=1}^{n}[c_i\Delta x_i + (W_i - u_i\Delta x_i)\tan(\phi_i)]\frac{\sec\alpha_i}{1+\tan(\phi_i)\tan\alpha_i/FOS}}{\sum_{i=1}^{n}(W_i\tan\alpha_i)} \tag{2}$$

where W_i is the weight of the ith slice, α_i is the tangential angle of the base of the ith slice, Δx_i is the ith slice width, c_i is the cohesion of the soil on the base of the ith slice, u_i is the pore water pressure at the base of the ith slice, and ϕ_i is the friction angle of the soil at the base of the *ith* slice. To obtain the minimum FOS of a slope, either a trial and error or an optimisation technique must be implemented [6].

In contrast with deterministic analyses, where the soil properties are characterised as fixed values, probabilistic analyses consider the uncertainty of soils within a slope. In a probabilistic analysis, slope stability is evaluated through considering the variation of soil properties, allowing the user to predict the probability of failure and reliability index. By definition, the reliability index (β) can be expressed as:

$$\beta = \frac{E[g(X)]}{\sigma[g(X)]} \tag{3}$$

$$g(X) = g(X_1, X_2, \ldots, X_n) \quad \text{for } i = 1 \text{ to } n$$

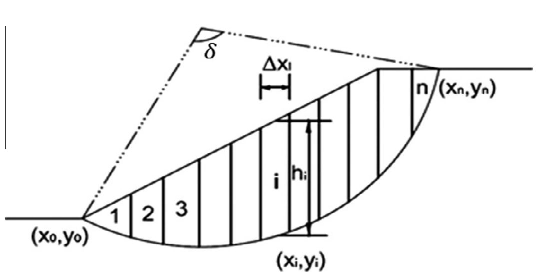

Fig. 1. Terms used to describe slip surface geometry.

in which $E[g(X)]$ is the mean value and $\sigma[g(X)]$ is the standard deviation of the limit state function $g(X)$. This equation can be used to evaluate the reliability at a design point, e.g. the reliability at a known slip surface. Hasofer and Lind [7] proposed an invariant approach to solve for the reliability index by transforming the random variables (X) into standardised normal variables ($\overline{X}$):

$$\overline{X_i} = \frac{X_i - E[X_i]}{\sigma[X_i]} \quad (i = 1, 2, \ldots, N) \tag{4}$$

So in a reduced variable space, the limit state function can be rewritten as:

$$g(\overline{X}) = g(\overline{X}_1, \overline{X}_2, \ldots, \overline{X}_n) \tag{5}$$

The limit state function defined by $(\overline{X}) = 0$, separates the safe region from the unsafe region as shown in Fig. 2. The minimum distance from this surface to the origin is the reliability index and can be calculated by Eq. (6).

$$\beta = \min_{\overline{X}\in\psi}\{\overline{X}\overline{X}^T\}^{1/2} \tag{6}$$

A number of researchers have successfully used this definition to determine the reliability index of slopes [8–12].

3. Solving multi failure mechanisms using a particle swarm method

3.1. Problem definition

This paper presents a method to find not only the slip surface with the minimum reliability index, but all possible discrete failure modes which are within a certain tolerance of the critical slip surface. This is achieved by using a modified particle swarm optimisation model which is discussed in detail later. There are a number of important reasons for determining additional failure modes; (1). The global minimum reliability index may be shared by several distinct slip surfaces. i.e. many slip surfaces may have the same probability of failure, however traditional analyses will only allow for one minimum slip surface. If these slip surfaces are located in different regions of the slope, they will not be accounted for by traditional methods. (2). When rehabilitating a slope it is necessary to find all regions below a certain threshold which require reinforcement, in this situation the global minimum is less important than the potential failure zones. (3). A slope could be susceptible to more than one failure mode simultaneously i.e. a large slope may

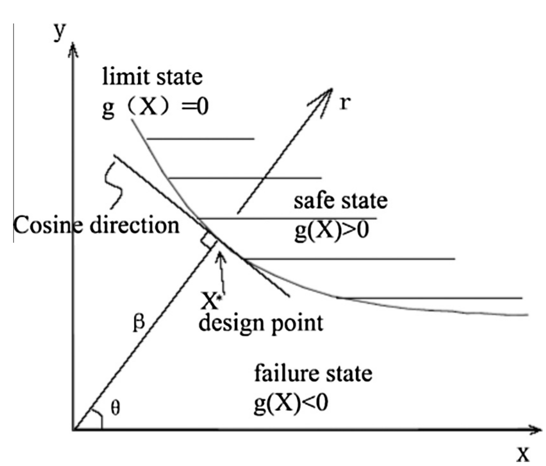

Fig. 2. Limit state and reliability index in orthogonal and polar coordinates, where the reliability index (β) is the minimum value of the radial distance (r) from the origin to the limit state function.