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A meshless method for axisymmetric problems in continuously nonhomogeneous saturated porous media

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ABSTRACT

A meshless method based on the local Petrov–Galerkin approach is proposed to analyze 3-d axisymmetric problems in porous functionally graded materials. Constitutive equations for porous materials possess a coupling between mechanical displacements for solid and fluid phases. The work is based on the u–u formulation and the incognita fields of the coupled analysis in focus are the solid skeleton displacements and the fluid displacements. Independent spatial discretization is considered for each phase of the model, rendering a more flexible and efficient methodology. Both displacements are approximated by the moving least-squares (MLS) scheme. The paper presents in the first time a general meshless method for the numerical analysis of axisymmetric problems in continuously nonhomogeneous saturated porous media. Numerical results are given for boreholes in continuously nonhomogeneous porous medium with prescribed misfit and exponential variation of material parameters in the excavation zone.

1. Introduction

Poroelasticity belongs to the continuum mechanics with two or more phases. The one-dimensional theory of the consolidation of a water saturated elastic porous geomaterial was first developed by Terzaghi [1]. Later Biot [2] formulated a theory for multidimensional problems of porous materials saturated by a viscous fluid. The generalized three-dimensional theory of poroelasticity in anisotropic porous materials has been developed by Biot [3] too. Theory of poroelasticity has been successfully applied in the study of variety of problems in geomechanics, biomechanics, materials engineering, environmental geomechanics and energy resource recovery from geological formations [4,5]. The extension to a nearly saturated poroelastic material has been presented by Aifantis [6] and Wilson and Aifantis [7] for the quasi-static case. The dynamic extension of Biot's theory to three phases has been published by Vardoulakis and Beskos [8]. A state of the art overview on the theory of dynamic poroelasticity, its numerical approximation, and applications may be found in Schanz' review paper [5].

Since the coupled differential equations are generally difficult to solve exactly, it appears that numerical approaches have to be adopted to attain solutions. Analytical methods are restricted to simple boundary value problems. A nice review is given by Selvadurai [9]. Despite the universality and great success of the finite and boundary element methods in their applicability even to multi-field problems, there are some restrictions leading to exclusion of the finite elements with equal order interpolation for pressure and displacements in poroelastic problems [10–13]. The dynamic Green's function of homogeneous poroelastic half-plane has been derived by Senjuntichai and Rajapakse [14] and later applied to a vertical vibration of an embedded rigid foundation in a poroelastic soil [15]. The boundary element method (BEM) has been developed for transient and time harmonic analysis of dynamic poroelasticity problems [16]. Later a simple BEM formulation for poroelasticity via particular integrals has been developed by Banerjee [17]. Time domain BEM has been applied for axisymmetric quasi-static poroelasticity [18]. Dynamic Green's functions for poroelastic and layered poroelastic half-spaces have been derived in [19] and [20]. In general, material coefficients in poroelasticity are anisotropic [21] and spatially variable.

Axisymmetric problems have received considerable attention in the past due to their close relevance to geotechnical and rock testing methods such as uni-axial and tri-axial compression tests, double-punch tests and point load strength tests. In addition, stress analysis of cylinders is also relevant to applications involving bio-medical and mechanical engineering. A cylindrical borehole drilled in a soil/rock medium is commonly found in the petroleum industry. Stability of borehole is important because it is the one of major

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problems in oil and gas industries. In the past, the classical theory of elasticity has been used extensively to analyze various elastostatic and elastodynamic problems involving cylinders and boreholes [22]. However, geological materials are normally two-phase materials consisting of a solid skeleton with voids filled with water. Such materials are commonly known as poroelastic materials and widely considered as much more realistic representation for natural soils and rocks than ideal elastic materials [23]. The governing equations for a poroelastic material undergoing axisymmetric deformations are given by Rice and Cleary [24]. A misfit of the radial displacement with respect to the borehole diameter is considered here. A cylindrical borehole in a poroelastic medium with consideration of excavation disturbed zone is considered. Shear modulus and permeability coefficient are assumed to be continuously non-homogenous in radial direction for the disturbed zone. Vrettos [25] derived Green's functions for vertical point load on a non-homogeneous medium with an exponential variation of the shear modulus decreasing with depth.

In spite of the great success of domain and boundary discretization methods for the solution of general boundary value problems, there is still a growing interest in the development of new advanced computational methods. The finite element method (FEM) can be successfully applied to problems with an arbitrary variation of material properties by using special graded elements. In commercial computer codes, however, material properties are considered to be uniform within each element. In recent years, meshless formulations are becoming popular due to their high adaptability and easier preparation of input and output data in numerical analyses. The moving least squares (MLS) approximation is generally considered as one of many schemes to interpolate discrete data with a reasonable accuracy. The order of continuity of the MLS approximation is given by the minimum between the orders of continuity of the basis functions and that of the weight function. Thus, continuity can be tuned to a desired value. In conventional discretization methods, however, the interpolation functions usually result in a discontinuity in the secondary fields (gradients of primary fields) on the interfaces of elements. For modeling coupled fields the approach based on piecewise continuous elements can bring some inaccuracies. Therefore, a model which is based on C^1-continuity, such as the meshless method, is expected to be more accurate than conventional discretization techniques. A drawback of meshless methods is higher CPU time compared to regular FEM. However, this drawback can be overcome. Recently, the authors [48–51] have developed a modified MLPG formulation, where Taylor series expansions and analytical integrations over the local sub-domains in two-dimensional elastodynamics are applied.

A variety of meshless methods can be derived from a weak-form formulation either on the global domain or on a set of local subdomains. In the global formulation, background cells are required for the integration of the weak-form. In methods based on local weak-form formulation, on the other hand, no background cells are required. The meshless local Petrov–Galerkin (MLPG) method is a fundamental base for the derivation of many meshless formulations, since the trial and test functions can be chosen from different functional spaces [26–29]. The MLPG method with a Heaviside step function as the test function [29] has been successfully applied to solve various 3-d axisymmetric problems [30–32]. The MLPG has been successfully applied to porous problems [33,52,53]. Many meshless formulations in poroelastic media have been applied to analyze consolidation problems [54–57]. The MLPG has been applied also to dynamic poroelastic problems, however up to day only as two-dimensional analyses [33,52]. In all early published papers based on the meshless formulations homogeneous material properties are considered. A meshfree algorithm based on the Galerkin approach is proposed for the fully coupled analysis of flow and deformation in unsaturated poroelastic media by Khoshghalb and Khalili [34]. Temporal discretization is achieved there using a three-point approximation technique with second order accuracy. Sheu [35] analyzed the prediction of probabilistic settlements with the uncertainty in the spatial variability of Young's modulus to illustrate the preliminary development of a spectral stochastic meshless local Petrov–Galerkin (SSMLPG) method. Generalized polynomial chaos expansions of Young's moduli and a two-dimensional meshfree weak–strong formulation in elasticity are combined to derive the SSMLPG formulation.

In the present paper, the MLPG is developed for an axisymmetric 3D boundary value problem in a porous material with continuously varying material properties. It is the first meshless application to such a problem. Because of the axial symmetry, the analyzed domain is the cross-section of the considered body with the plane involving the axis of symmetry. Both governing equations for the balance of momentum in solid and fluid phases are satisfied in a weak form on small fictitious subdomains in the present paper. Nodal points are introduced and spread on the analyzed domain and each node is surrounded by a small circle for simplicity; but in general, it can be of an arbitrary shape. The spatial variations of the displacements in solid and fluid phases are approximated by the moving least-squares scheme [36,37]. After performing the spatial integrations, one obtains a system of ordinary differential equations (ODE) for temporal variations of certain nodal unknowns. The backward difference method is applied for the approximation of "velocities" and the Houbolt method [38] is applied for the accelerations in the ODE. The influence of the material gradation on displacements and stresses in porous medium around the borehole is investigated.

2. Local boundary integral equations

In Biot's theory a fully saturated material is considered, i.e., an elastic skeleton with a statistical distribution of interconnected pores is modeled. The porosity is denoted by

$$\phi = \frac{V^f}{V} \tag{1}$$

where V^f is the volume of the interconnected pores contained in a sample of bulk volume V. The sealed pores are considered as part of the solid. Full saturation is assumed leading to $V = V^f + V^s$ with V^s the volume of the solid. There are several possibilities to write governing equations for porous materials:

(i) To use the solid displacement u_i^s and the fluid displacement u_i^f with six (four) unknowns in 3-d (2-d) problems [39].

(ii) Alternatively the solid displacement u_i^s and the seepage displacement w_i ($w_i = \phi(u_i^f - u_i^s)$) with also six (four) unknowns in 3-d (2-d) can be used [40]. Beside the seepage displacement also sometimes the seepage velocity, i.e., the time derivative of w_i is applied.

(iii) A combination of the pore pressure p and the solid displacement u_i^s with four (three) unknowns in 3-d (2-d) can be established. As shown by Bonnet [41], this choice is sufficient.

In the present paper we use the first approach based on solid and fluid displacements. If the constitutive equations are formulated for the elastic solid and the interstitial fluid, a partial stress formulation is obtained [2,3]

$$\sigma_{ij}^s = 2G\varepsilon_{ij}^s + \left(K - \frac{2}{3}G + \frac{Q^2}{R}\right)\varepsilon_{kk}^s\delta_{ij} + Q\varepsilon_{kk}^f\delta_{ij}, \tag{2}$$

$$\sigma^f = -\phi p = Q\varepsilon_{kk}^s + R\varepsilon_{kk}^f, \tag{3}$$