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Semi-analytical solution to one-dimensional consolidation for viscoelastic unsaturated soils

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ABSTRACT

This paper presents a semi-analytical solution to one-dimensional consolidation of viscoelastic unsaturated soils with a finite thickness under oedometric conditions and subjected to a sudden loading. The solution is obtained by using Lee's correspondence principle based on the semi-analytical solution to one-dimensional consolidation of elastic unsaturated soils. The boundary contains the top surface permeable to water and air and the bottom impermeable to water and air. A typical example is given to show the evolution of excess pore-air and pore-water pressures as well as the total degree of consolidation of the soil layer with time for different ratios of air–water permeability coefficient, elastic modulus and viscoelastic coefficient. The one-dimensional consolidation behavior of viscoelastic unsaturated soil is discussed according to the semi-analytical solution. These results contribute to a better understanding of the consolidation behavior of viscoelastic unsaturated soils.

1. Introduction

Research on the consolidation behavior of unsaturated soils began in the 1960s. In the past five decades, a considerable amount of research on the consolidation theory for unsaturated soils has been conducted with many research results being accumulated, and great progress being made. Typical ones are due to Scott [1], Blight [2], Barden [3] and Fredlund and Hasan [4]. Among them, the consolidation theory of Fredlund and Hasan is the most popular. However, the above mentioned research works focus on elastic unsaturated soils, whereas in practice real soils exhibit more complex rheological properties (time-dependent creep for example) which has important impacts on engineering designs. The viscoelastic model was first introduced to consolidation theory for saturated soils by Tan [5] in the 1950s. Research works on this topic were then developed and some progress were made in subsequent years by Chen [6], Gibson and Lo [7], Lo [8], Xie and Liu [9], Leo and Xie [10], among others. However, up to this date, related research works on the consolidation theory of viscoelastic unsaturated soils are scarce.

Qin et al. [11]obtained an analytical solution of one-dimensional consolidation based on the formulation of Fredlund and Hasan [4] for linearly elastic unsaturated soils, subjected to a vertical step loading and zero radial strain in the horizontal directions (i.e. oedometric conditions). In their work, the top boundary surface is permeable while that of the bottom is impermeable to water and air. Shan et al. [12] provided exact solutions using both homogeneous and nonhomogeneous boundary conditions. Zhou et al. [13] presented a simple analytical solution to Fredlund and Hasan's one-dimensional consolidation theory for unsaturated soils. Ho et al. [14] introduced an exact analytical solution for governing equations of Fredlund and Hasan's one-dimensional consolidation in unsaturated soil stratum using the techniques of eigenfunction expansion and Laplace transformation. All the above solutions are for linearly elastic unsaturated soils. When the solution for an elastic body being known, the solution to the same problem for a viscoelastic body can be obtained by Lee's correspondence principle [15].

In this paper, based on the solution to one-dimensional consolidation of elastic unsaturated soils in [11], Lee's correspondence principle is adopted to study one-dimensional consolidation of viscoelastic unsaturated soils.

2. Constitutive equation

An unsaturated soil layer is considered with infinite horizontal extent and a finite thickness H subjected to a vertical step loading q. Radial strains are null in the horizontal directions (i.e. oedometric conditions). A representative soil element of volume $dV = 1 \times 1 \times dz$ with one-dimensional water and air flow in the z direction is shown in Fig. 1.

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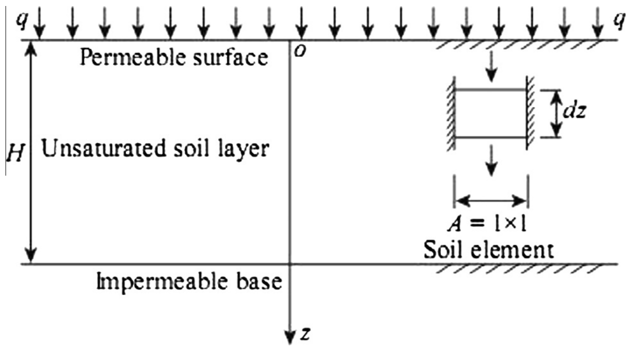

Fig. 1. One unsaturated soil layer with a free boundary at the top surface and an impermeable boundary at the bottom.

In this paper, the Merchant viscoelastic model [8] is adopted as the constitutive model for unsaturated soils, which consists of an elastic body L_0 in series with a Kelvin body. The latter is composed of two bodies in paralleled: an elastic body L_1 and a viscous body N, as shown in Fig. 2. A few steps of computations show that the Merchant model is described by the following constitutive equation:

$$\sigma + \frac{\eta}{E_0 + E_1}\frac{d\sigma}{dt} = \frac{E_0 E_1}{E_0 + E_1}\varepsilon + \frac{\eta E_0}{E_0 + E_1}\frac{d\varepsilon}{dt} \tag{1}$$

In the above equation σ is the stress and ε is the strain, E_0 is the stiffness coefficient of the elastic body L_0; E_1 and η are respectively the stiffness and viscosity coefficients of the elastic and viscous components of the Kelvin body.

Applying the Laplace transform to Eq. (1) we obtain:

$$V(s) \equiv \frac{\tilde{\varepsilon}(s)}{\tilde{\sigma}(s)} = \frac{1}{E_0} + \frac{1}{E_1 + \eta s} \tag{2}$$

where s is the conjugate variable of t in the Laplace transform.

3. Derivation of semi-analytical solution

3.1. Governing equation for water phase

The permeability of water in the unsaturated soil is assumed constant during consolidation for a preliminary study, which is a simplifying assumption that may influence the results, but needed to be adopted to obtain the solution for the exiting complex differential equations. Based on the continuity equation and Darcy's law applied to the pore water, we have [4]

$$\frac{\partial\left(\frac{V_w}{V_0}\right)}{\partial t} = \frac{k_w}{\gamma_w}\frac{\partial^2 u_w}{\partial z^2} \tag{3}$$

where V_0 is the initial soil volume; V_w is the volume of water phase; k_w is the coefficient of water permeability in unsaturated soils; u_w is the excess pore-water pressure due to external load and γ_w is the unit weight of water phase.

The Laplace transform applied to Eq. (3) leads to:

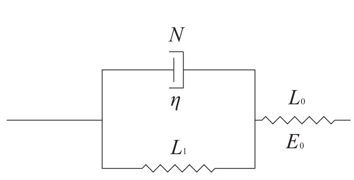

Fig. 2. The Merchant model.

$$s\left(\frac{\tilde{V}_w}{V_0}\right) = \frac{k_w}{\gamma_w}\frac{\partial^2 \tilde{u}_w}{\partial z^2} \tag{4}$$

Volume change of the water phase in unsaturated soils is described by the following equation due to Fredlund and Hasan [4]:

$$\frac{\partial\left(\frac{V_w}{V_0}\right)}{\partial t} = m_{1k}^w \frac{\partial(\sigma - u_a)}{\partial t} + m_2^w \frac{\partial(u_a - u_w)}{\partial t} \tag{5}$$

The coefficients m_{1k}^w and m_2^w account for changes in water volume due respectively to changes in net normal stress $\sigma - u_a$ and suction $u_a - u_w$; u_a is the excess pore-air pressure and σ is the total stress. The subscript k stands for K_0 loading condition without zero lateral strain.

Considering that the total stress σ is constant during consolidation, applying the Laplace transform to Eq. (5) gives the following equation on change of water volume in the transformed domain:

$$s\left(\frac{\tilde{V}_w}{V_0}\right) = \tilde{m}_{1k}^w\{s(-\tilde{u}_a) - (-u_a^0)\} + \tilde{m}_2^w\{s(\tilde{u}_a - \tilde{u}_w) - (u_a^0 - u_w^0)\} \tag{6}$$

where u_a^0 and u_w^0 are the initial excess pore-air and pore-water pressures, and

$$\tilde{m}_{1k}^w = -\frac{1}{E_{01k}^w} - \frac{1}{E_{1k}^w + \eta_{1k}^w s} \tag{7}$$

$$\tilde{m}_2^w = -\frac{1}{E_{02}^w} - \frac{1}{E_2^w + \eta_2^w s} \tag{8}$$

in which, E_{01k}^w and E_{1k}^w are the stiffness coefficients accounting for changes in net normal stress $\sigma - u_a$, respectively for the elastic body L_0 and the Kelvin body. Stiffness coefficients E_{02}^w and E_2^w are the corresponding stiffness coefficients accounting for suction changes. Finally, η_{1k}^w and η_2^w are the viscous coefficients of the Kelvin body accounting respectively for changes in net normal stress and suction.

Substituting Eq. (4) into Eq. (6) gives

$$\tilde{m}_{1k}^w\{s(-\tilde{u}_a) + u_a^0\} + \tilde{m}_2^w\{s(\tilde{u}_a - \tilde{u}_w) - (u_a^0 - u_w^0)\} = \frac{k_w}{\gamma_w}\frac{\partial^2 \tilde{u}_w}{\partial z^2} \tag{9}$$

Rearranging Eq. (9) leads to the following equation:

$$s\tilde{u}_w = -\tilde{C}_v^w \frac{\partial^2 \tilde{u}_w}{\partial z^2} + s\tilde{C}_w \tilde{u}_a + u_a^0 \tilde{C}_w + u_w^0 \tag{10}$$

where

$$\tilde{C}_w = \frac{\tilde{m}_{1k}^w - \tilde{m}_2^w}{\tilde{m}_2^w} = \frac{1 - \tilde{m}_2^w/\tilde{m}_{1k}^w}{\tilde{m}_2^w/\tilde{m}_{1k}^w} \tag{11}$$

$$\tilde{C}_v^w = \frac{k_w}{\gamma_w \tilde{m}_2^w} \tag{12}$$

3.2. Governing equation for air phase

The coefficient of air permeability k_a is assumed constant during consolidation for a preliminary study, which is a simplifying assumption that may influence the results, but needed to be adopted to obtain the solution for the exiting complex differential equations, and the pore-air behaves is assumed to an ideal gas, we have [4]

$$\frac{\partial\left(\frac{V_a}{V_0}\right)}{\partial t} = \frac{k_a RT}{g\bar{u}_a M}\frac{\partial^2 u_a}{\partial z^2} - \frac{n(1 - S_r)}{\bar{u}_a}\frac{\partial u_a}{\partial t} \tag{13}$$

where V_a is the volume of air phase. R is the universal gas constant, 8.314 J/(mol K). T is the absolute temperature, K. M is the average molecular mass of air phase, kg/mol; g is the gravitational