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Analysis of laterally loaded pile groups in multilayered elastic soil

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ABSTRACT

The paper presents a semi-analytical method of calculating the response of a pile group. The approach is based on tying the displacement at any point of the soil mass around a pile or group of piles to the displacements experienced by the piles themselves. This is done by multiplying the pile displacements by decay functions. Application of the principle of minimum potential energy and calculus of variations to the resulting displacement field formulation leads to the differential equations for the soil and piles. Solution of these differential equations using finite differences and the method of eigenvectors leads to the desired displacement field in the soil and deflection profiles of the piles. The method produces displacement fields that are very close to those produced by the finite element method at a fraction of the cost. To illustrate the ease of application of the method, it is then used to prepare pile group efficiency charts for some typical soil modulus profiles.

1. Introduction

1.1. The problem of the laterally loaded pile group

It is common for a single pile to not have enough capacity to sustain a structural load (as from a column in a frame structure) on its own, so pile groups capped by a reinforced concrete pile cap are very common in foundation engineering solutions. The present paper proposes a new solution for the problem illustrated in Fig. 1, which addresses the lateral loading of a group of n_p piles connected at the top by a rigid cap and installed in a soil profile consisting of n_{total} layers. The main aim of any analysis of this problem is to relate the total displacement of the pile cap to the load applied on it. Also of interest are estimates of the displacements that develop in the surrounding soil and what the internal forces in the piles are.

Horizontal forces may be due to wind, waves, traffic or seismic loadings. These loads are, in the end, transferred to the piles supporting the structures. The horizontal forces get transmitted to the pile cap and then to the top of individual piles as concentrated forces and/or moments. One of the challenging aspects of pile design under lateral loads is to determine the fraction of the total loading that gets distributed to each pile. Given the range of structures subject to significant lateral loading, there has been considerable research on the problem of laterally-loaded piles, an indicator of the importance of the problem and an indicator also of the lack of a definitive, satisfactory solution to the problem. The literature on the topic is reviewed next.

1.2. Analysis and design approaches

1.2.1. Subgrade reaction method

Analysis of laterally loaded piles was initially based on the concept of representing soil by discrete springs using Winkler's beam on elastic foundation approach [1]. However, this approach was modified to account for plastic deformation of soil (which starts at very small strains) by incorporating non-linearity in the springs [2,3]. Further development of this concept led to the p–y method, which is widely used today.

In the p–y method, load–displacement (p–y) curves are associated with different depths along the pile, and the pile deflections are calculated iteratively using the so-called p–y curves [4–8]. For routine design, the p–y method is the method of choice, as finite element analyses are too costly, but it suffers from limitations [9–16].

1.2.2. Continuum approach

The continuum approach assumes the pile as embedded in an elastic continuum. Classical work (e.g., [17,18]) on the problem of the laterally loaded pile group has relied on analytical and numerical elastic techniques and principles, often including the principle of superposition, to solve it. The variational approach has been used to set up the boundary value problem for a pile loaded laterally in an elastic medium with some assumptions on the form of the displacement field; analytical or numerical solution of the

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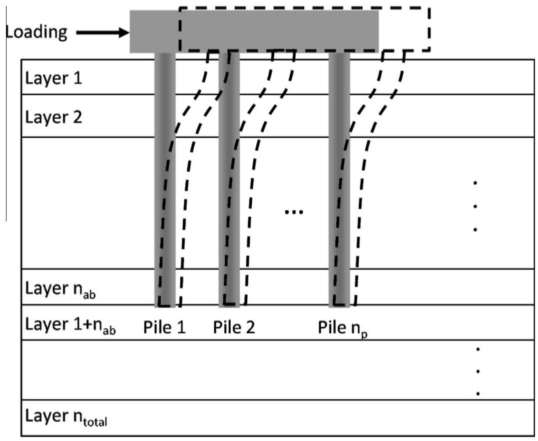

Fig. 1. A laterally loaded pile group in a multi-layered soil profile.

problem then led to the pile displacements for different boundary conditions [19,20]. Similarly, analyses of a laterally loaded pile installed in multi-layered soil were done by assuming mathematical forms for the displacement field in the soil and minimizing potential energy for the pile–soil system [21–23].

There has also been considerable work on use of numerical methods, particularly the finite element method [24–29] to study the laterally loaded pile group problem. The major advantage of numerical techniques is their flexibility in adapting to different geometries, boundary conditions and constitutive relationships. However, they are mostly problem-specific and computationally intensive, requiring, in addition, both sufficient experience on the part of the analyst and time to properly set up the analysis. In contrast, linear elastic methods cannot be applied in practice without a measure of judgment, but provide insights into pile load response and establish the conceptual basis for more realistic methods of analysis.

1.3. Pile group analysis and goals of the present paper

Field and model experiments have shown that pile group response to lateral loads is extremely complex and depends on many factors, including loading conditions, pile end restraints, pile arrangement and spacing, and the stiffness of each pile relative to the other piles and the soil [30–37]. In early research, displacement and load distribution among the piles were determined considering the effect of soil as elastic springs [38–40]. The most commonly used method of analysis today is the *p*–*y* multiplier technique. Based on full-scale tests of pile groups, it is known that, all other things being equal, a pile group deflects more than an isolated pile loaded to a load equal to the average load per pile in the group [25,41] because the soil stiffness is reduced due to the overlapping of deformation zones of neighboring piles. The *p*–*y* multiplier approach accounts for this by using multipliers (with value less than one) to reduce the ordinates of the single-pile *p*–*y* curve so that it can then be applied to each individual pile in the group. These multipliers have typically been back-calculated from experimental and numerical results [42–45,36,46,47]. The multiplier values are problem-specific, and there is no rational method available that can be used to predict the multipliers in a generalized way.

An evaluation of the different available methods [48] revealed that no single method was adequate in analyzing all aspects of pile groups, such as pile–soil–pile interaction, position and spacing of piles, and the relative stiffness of the piles. Deficiencies of the different methods have been reported by other researchers as well [49–51]. The existing methods for analyzing the laterally-loaded pile problem suffer from one or more of the following limitations: (1) need for important assumptions and approximations, (2) analyses that are difficult to use in practice or that do not provide much insight into the problem or (3) continued reliance on representation of the soil by springs. This paper presents an analysis of laterally-loaded pile groups in multi-layered, elastic soil media. The analysis is based on the assumption that the displacements at points in the soil is a function of the displacements of each pile in the group but the analysis does not rely on the superposition principle, which means there are no restrictions on its future use to a material that is not linear elastic. With the formulation of the displacement field established, the principle of minimum potential energy (or, more generally, the principle of virtual work) can be used to set up the formulation. The analysis is equally applicable to single piles (by simply making the number of piles equal to 1). The analysis has the strengths that it is based on proper physics; it is easy to use once it has been coded; and, being a continuum mechanics-based solution, it establishes the basis for future improvements, including use of more realistic constitutive models.

2. Theoretical framework

2.1. Displacement, strain and stress fields

The displacement $\{u(x,y,z)\}$ at any point in the soil mass around a pile group is linked to the displacement experienced by each pile in the group. The lateral component of $\{u(x,y,z)\}$ in the soil can then be expressed as the summation, for all n_p piles in the group, of the product of the lateral displacement $w_i(z)$ of pile *i* by a dissipation or decay function $f_i(x,y)$ associated with pile *i*. Each of these n_p decay functions varies between 1 at the location of the specific pile the decay function is associated with and zero both at the locations of all the other piles and at an infinite distance from the pile group (in practical terms, at the boundaries of the domain used to approximate the soil half space). One of the important advantages of this approach is that this assumption on soil displacement can be made regardless of the constitutive model used for the soil, i.e., it is not an application of the superposition principle, with the important implication that the approach is not restricted to an elastic model of the soil. The displacement field may be assumed more or less complex, making it more or less realistic.

In this paper, we assume a form for the displacement field in terms of Cartesian coordinates and assume a linear-elastic model for the soil. The simplest possible displacement function around a pile group is given by:

$$\begin{aligned} u_x &= \sum_{i=1}^{n_p} w_i(z) f_i(x,y) \\ u_y &= u_z = 0 \end{aligned} \tag{1}$$

where $f_i(x,y)$ is the decay function that attenuates the displacement $w_i(z)$ induced by the *i*th pile across the domain. Eq. (1) applies regardless of the shapes of the cross sections of the piles and, as we will show, produces excellent results despite its simplicity. Differentiation of (1) leads to the infinitesimal strain field (positive if contractive):

$$\varepsilon_{kl} = -\frac{1}{2}(u_{k,l} + u_{l,k}) \tag{2}$$

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{xy} \\ \varepsilon_{xz} \\ \varepsilon_{yz} \end{bmatrix} = \begin{bmatrix} -\frac{\partial u_x}{\partial x} \\ -\frac{\partial u_y}{\partial y} \\ -\frac{\partial u_z}{\partial z} \\ -\frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right) \\ -\frac{1}{2}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right) \\ -\frac{1}{2}\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right) \end{bmatrix} = \begin{bmatrix} -\sum_{i=1}^{n_p} w_i(z) \frac{\partial f_i(x,y)}{\partial x} \\ 0 \\ 0 \\ -\frac{1}{2}\left(\sum_{i=1}^{n_p} w_i(z) \frac{\partial f_i(x,y)}{\partial y}\right) \\ -\frac{1}{2}\left(\sum_{i=1}^{n_p} \frac{dw_i(z)}{dz} f_i(x,y)\right) \\ 0 \end{bmatrix} \tag{3}$$