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An elastoplastic model with combined isotropic–kinematic hardening to predict the cyclic behavior of stiff clays

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ABSTRACT

This paper presents a kinematic hardening model for describing some important features of natural stiff clays under cyclic loading conditions, such as closed hysteretic loops, smooth transition from the elastic behavior to the elastoplastic one and changes of the compression slope with loading/unloading loops. The model includes two yield surfaces, an inner surface and a bounding surface. A non-associated flow rule and a kinematic hardening law are proposed for the inner surface. The adopted hardening law enables the plastic modulus to vary smoothly when the kinematic yield surface approaches the bounding surface and ensures at the same time the non-intersection of the two yield surfaces. Furthermore, the first loading, unloading, and reloading stages are treated differently by applying distinct hardening parameters. The main feature of the model is that its constitutive equations can be simply formulated based on the consistency condition for the inner yield surface based on the proposed kinematic hardening law; thereby, this model can be easily implemented in a finite element code using a classic stress integration scheme as for the modified Cam Clay model. The simulation results on the Boom Clay, natural stiff clay, have revealed the relevance of the model: a good agreement has been obtained between simulations and the experimental results from the tests with different stress paths under cyclic loading conditions. In particular, the model can satisfactorily describe the complex case of oedometric conditions where the deviator stress is positive upon loading (compression) but can become negative upon unloading (extension).

1. Introduction

It is well known that the stress–strain curves of soils under cyclic loading conditions show hysteresis loops with gradual accumulation of permanent strain. Indeed, various experimental results from isotropic compression tests, drained triaxial shear tests and oedometer tests on natural stiff clays (Boom Clay and Ypresian Clays, for instance) with several unloading/reloading cycles show marked hysteresis loops [1–3]. Natural stiff clays also exhibit smooth transition from elastic to elastoplastic compression (progressive stiffness degradation with strain) for either loading or unloading/reloading stages. In addition, experiments show another important characteristic regarding the compression slope in the reloading process before reaching $\sigma'_{v\max}$ which is the maximum vertical stress applied before unloading. This compression slope in the reloading process varies significantly from one loop to another. Nguyen [4] concluded that this slope increases with $\sigma'_{v\max}$. These features must be taken into account when developing constitutive models for the description of the mechanical behavior of this kind of clays under cyclic loading conditions.

Conventional critical state models for soils including the Modified Cam Clay model (MCC) can describe plastic strains in the normally consolidated state, but only elastic strains are produced during the subsequent unloading–reloading cycles within the yield surface. On the other hand, bounding surface models with radial mapping rule proposed in the 1980s (see e.g. [5,6]), where the current plastic modulus varies with the distance between the stress state and its image point on the bounding surface, can successfully describe some important features of natural stiff clays such as the smooth transition from elastic to elastoplastic states as well as the softening behavior. However, this kind of models gives open hysteresis loops during unloading–reloading stages and cannot describe the cyclic loading behavior realistically, since no plastic strain is generated during the unloading process. To overcome this deficiency, attempts were made by some researchers (e.g. [7,8]) by applying the generalized plasticity concept proposed by

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Zienkiewicz and Mroz [9]. It is assumed that plastic strain is produced even by a stress increment directed toward the inside of the yield surface, the stress point always lying on the inner yield surface. With the generalized plasticity concept, a gradual strain accumulation with closed hysteresis loops can be simulated for the loading compression side (the deviator stress is positive). However, it fails for the extension side (negative deviator stress): an inflection appears in the stress–strain curve upon unloading along a straight stress path [10] as illustrated in Fig. 1. It is assumed that the loading yield surface has an ellipse shape as in the MCC model (see Fig. 1(a)). During the unloading process (path 1–2), the loading surface shrinks from f_{l1} to f_{l2} with the size parameter decrease from p'_{c1} to p'_{c2}. A negative plastic strain increment is generated for this loading path. Note that point 3 is the instantaneous point at which the increment of the yield surface size is null (i.e. $dp'_c = 0$) and after point 3 the yield surface size starts to expand after having shrunk between points 1 and 3. At point 3, just elastic strain is produced leading to an inflection point (see Fig. 1(b)). After point 3, the loading surface expands along p' axis passing through the origin of the stress space with a positive plastic strain increment generated. Indeed, the predicted behavior that a negative plastic strain increment generated along path 1–3 but a positive plastic strain increment along path 3–4 is difficult to admit physically. Hence, this kind of models is not suitable for simulating the oedometer tests where negative deviator stress occurs in the unloading process.

An important development in the constitutive modeling for the cyclic loading behavior is the introduction of kinematic hardening mechanism by Mroz [11]. The ‘Bubble’ model by Al-Tabbaa [12] was developed within the framework of kinematic models. In this model, a kinematic yield surface (namely bubble surface) is defined, which is allowed to translate and expand or contract within the conventional yield surface (namely bounding surface). The formulation of such a kinematic hardening model is mainly centered on the translation rule and the hardening function: the former is used to control the movement and interaction of the two surfaces and the latter is defined to describe the variation of the plastic modulus. This model can reproduce a closed hysteretic loop under a complete cyclic loading with deviator stress being from positive values to negative values. However, it should be pointed out that in the bubble model, the plastic modulus of the current stress state is not formulated by considering the consistency condition of the kinematic yield surface, but given by an interpolation function depending on the distance from the current stress point to the bounding surface. In addition, the kinematic hardening models are still not widely used to describe the cyclic behavior of natural stiff clays, such as Boom Clay.

In this study, a kinematic hardening model for modeling the cyclic behavior of natural stiff clays is developed. Basically, the developed model has a structure similar to that of bubble models. However, instead of defining an interpolation function for the plastic hardening modulus, a kinematic hardening law associated with the kinematic yield surface is defined, enabling the plastic modulus to vary smoothly along a plastic loading process. With a translation rule incorporated, this hardening law ensures that the two yield surfaces do not intersect but tend to coincide at the current stress point. Thereby, the constitutive equations can be simply obtained based on the consistency condition of the kinematic yield surface which is equivalent to the classic yield surface. Hence, in the numerical implementation, all the features of the stress integration schemes for classic elastoplastic models can be applied. This allows the model to be employed easily in the analysis of geotechnical problems. Furthermore, different model parameters for describing hardening rate in the loading/unloading/reloading processes are introduced enabling the cyclic loading behavior of natural stiff clays to be described in a flexible fashion. Also, a non-associated flow rule is adopted in order to properly describe the dilatancy behavior. The simulation of a series of tests on natural Boom Clay along different loading paths including the complex oedometric path show the relevance of the model proposed.

2. Model description

For the sake of simplicity, it is assumed that the soil behavior is isotropic. This hypothesis will clearly limit the proposed constitutive model if inherent and induced anisotropy is to be described. However, the extension of this model to account for anisotropy is feasible by incorporating new plastic mechanisms such as a rotational hardening rule associated with an inclined surface as proposed for instance by Wheeler et al. [13] or by explicitly incorporating soil fabric and its evolution in the model, as an extension of the work of Rouainia and Wood [14] or Baudet and Stallebrass [15], for instance, or as done more recently by Gao et al. [16], who effectively tackled the issue of anisotropy.

In what follows, the constitutive model is developed and formulated in the conventional triaxial conditions, i.e., when two effective principal effective stresses are equal ($\sigma'_2 = \sigma'_3$). The model can be extended to a general stress state easily if the soil parameters are properly determined as pointed out by Mroz et al. [17]. By defining the compressive stresses and strains as positive, the mean

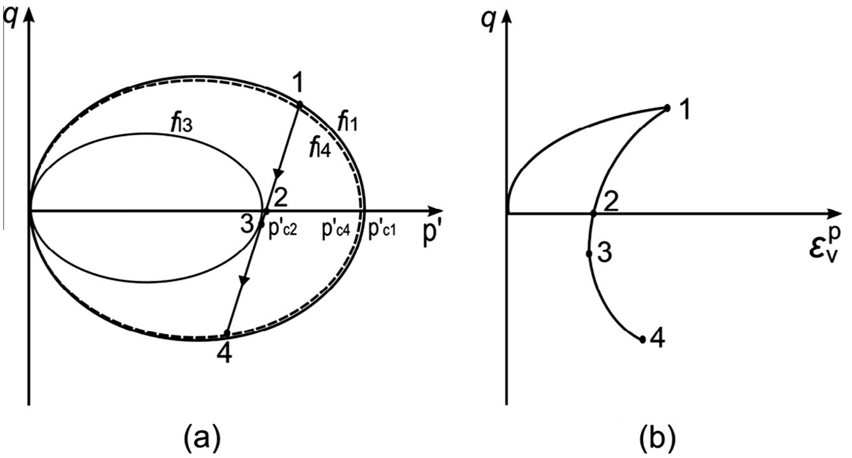

Fig. 1. Problems related to the bounding surface models with isotropic hardening law considering plastic strain in the unloading process: (a) the stress path in the (p', q) plane and (b) the stress-volumetric plastic strain curve.