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A two-grid search scheme for large-scale 3-D finite element analyses of slope stability

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ABSTRACT

It is well known that the trial process for seeking the safety factor in the shear strength reduction finite element method (SSRFEM) is quite expensive, particularly for large 3-D slope stability analyses. The search algorithm for the safety factor is crucial to the entire solution process for the shear strength reduction finite element method, but few studies have attempted to exploit it. Among search algorithms, the commonly used bracketing and bisection search has not been fully optimised. Consequently, to improve the search scheme for the safety factor associated with the shear strength reduction finite element method, two strategies are suggested. First, a generalised bisection search algorithm is proposed to reduce the possibility of encountering non-convergence from a statistical point of view. To further improve the efficiency, a new two-grid scheme, characterised by a coarse mesh search and followed by a fine mesh search, is developed. Based on the drained or undrained analyses of the 3-D slope examples, the new search algorithm can markedly outperform the commonly used bisection search algorithms based on a single finite element mesh.

1. Introduction

Geotechnical stability is a classical and long-standing problem in the geotechnical field, and according to Sloan [1], four typical methods are generally used to perform a geotechnical stability analysis. Conventionally, slope stability analysis has resorted to limit-equilibrium methods (e.g. [2]), but the shear strength reduction finite element method (SSRFEM) has become appealing only recently following advances in computer technologies. The limit equilibrium methods (LEM) possess the advantages of simplicity and effectiveness, but the FE-based slope stability analysis methods may provide more information (such as the stress and deformation fields) apart from the safety factor. One of the earliest ideas for strength reduction is credited to Zienkiewicz et al. [3], but little attention had been given to it until the works by Ugai [4], Matsui and San [5], Dawson et al. [6], and Griffiths and Lane [7]. Because of these developments, the potential for the SSRFEM method has been widely exploited, but it was initially restricted to 2-D simulations (e.g. [8]). The 3-D finite element analysis, which had been hindered in past decades by the limitations of computer hardware and solution technologies, only recently became practical for geotechnical applications [9,10]. For example, Cai and Ugai [11], Zheng et al. [12], Griffiths and Marquez [13], Wei et al. [14], Huang and Jia [15] and Wei et al. [16] studied slope stability within the 3-D finite element framework. To satisfy rapidly growing demands, the commercial FEM software PLAXIS extends the facility of shear strength reduction from 2-D to 3-D simulations (e.g. [17]). According to these studies, the advantages of the SSRFEM method for slope stability over the limit-equilibrium approaches have been very well recognised. For instance, in the finite element analysis, no assumptions need to be made about the location or shape of the slip surface and the slice side forces versus the limit-equilibrium methods. Some classical elasto-plastic soil models, such as the Mohr–Coulomb model and the Drucker–Prager model, can be readily incorporated. The complicated geometry, boundary and loading conditions can be easily treated, and the stress/deformation fields can be progressively monitored. The most important benefit is that the SSRFEM method provides a more realistic analysis tool for complicated slopes, which explains why some researchers believe that when combined with the shear strength reduction (SSR) technique, the FEM is a powerful tool for slope stability analysis (e.g. [18]).

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Based on our experience, the problems associated with the SSR technique, including the high computational cost of the SSR technique and the very limited soil models that are appropriate for it, remain unsolved, which hinders its applications. Originally, the SSR technique was proposed for the Mohr–Coulomb and Drucker–Prager models, whereas other models, such as the Hoek–Brown model, were to be transformed to suit this technique (e.g. [19–21]). Hence, this work aims to improve the efficiency of the SSR technique to make it realistic and appealing for large-scale 3-D geotechnical applications, such as slope stability analysis.

When implementing the SSRFEM method, Smith and Griffiths [22] evaluated a series of trial strength reduction factors to seek the factor of safety (*FOS*). However, providing this sequence of factors can be difficult for inexperienced users. Hence, an initial search range for the *FOS* should be prescribed. Within the predefined search range, the question of how to search for the *FOS* effectively becomes crucial because the search algorithm can significantly influence the accuracy of the *FOS* and the entire computational cost. Among search algorithms, the bracketing and bisection search algorithm is most frequently used (e.g., [6,23–26]). Another search method, which was mentioned by Won et al. [27], is called the incremental refining search. These methods have been frequently utilised but have been subjected to further improvement because their inherent characteristics have not been fully exploited. For example, the nonlinear iterations' distribution in the search range is uneven; that is, the number of nonlinear iterations in the convergence range is usually small, while the number of nonlinear iterations in the non-convergence range is the maximum allowable number (such as 1000 or 500). Without considering this characteristic, the bracketing and bisection search method sets equal weights on the convergence and nonconvergence ranges. This paper is thus organised as follows. Section 2 briefly introduces the SSRFEM method using the Mohr–Coulomb material model. In Section 3, the bracketing and bisection search algorithm is reviewed first, and then a generalised bisection search algorithm is proposed with a bias towards the convergence range so the proposed search method can appropriately reduce the likelihood of encountering non-convergence. In Section 4, several schemes used for the preliminary search bound of the *FOS* are introduced, and a new two-grid scheme is emphasised. Section 5 introduces several 3-D slope examples upon which the newly proposed search algorithm is assessed, and the proposed algorithm is compared with the commonly used ones. Finally, concluding remarks are given in Section 6.

2. Shear strength reduction finite element method

In numerical simulations, geotechnical failures can be triggered by either an increase in the external loads or a decrease in the geomaterial strength, which lead to the incremental loading method and shear strength reduction method, respectively, as demonstrated in Fig. 1a and b. Compared with the incremental loading method, the shear strength reduction method has received a wider range of applications (e.g. [28]).

When the Mohr–Coulomb failure criterion is used, the shear strength is expressed as

$$\tau_f = c' + \sigma_n \tan\phi' \quad (1)$$

where the cohesion c' and the internal friction angle ϕ' are two effective shear strength parameters. With the SSR technique, the shear strength is decreased by reducing the values of the two shear strength parameters, that is,

$$c_r = c'/SRF_c \quad (2a)$$

$$\tan\phi_r = \tan\phi'/SRF_\phi \quad (2b)$$

where SRF_c and SRF_ϕ are the strength reduction factors corresponding to the cohesion and the internal friction angle, respectively, and it is usually assumed that $SRF_c = SRF_\phi = SRF$. When applying the shear strength reduction method to geotechnical problems such as slope stability analyses, three behaviours can be utilised to identify the occurrence of slope failure, namely, sudden substantial changes in the displacement of various marked nodes, a connected plastic shear band, and a non-equilibrium state. Because all three slope behaviours result in the non-convergence of the nonlinear iterative method, assessing the convergence or non-convergence of the nonlinear iterative method, which may be utilised independently or in conjunction with other behaviours/criteria, is an effective approach for judging slope stability in finite element analyses. Consequently, given the predefined maximum number of iterations *maxit_nl* and the convergence tolerance *tol_nl*, the non-convergence of the nonlinear iterative method is employed in the present study to identify slope failures, which must be carefully assessed to ensure that they are produced by a genuine geotechnical failure instead of artificial numerical effects (e.g. [1]). When an iterative linear solver is employed, the non-convergence of the nonlinear iterative method may be attributed to the non-convergence of the iterative linear solver. In that case, the direct linear solver can be employed to rule out this possibility. Evidently, *SRF* at the transition point from convergence to non-convergence can be defined as the *FOS*, that is, $FOS = SRF^f$.

When increasing the *SRF*, the transition point from convergence to non-convergence is defined as the *FOS*. The bisection algorithm was originally proposed for finding the roots of a nonlinear equation, $f(x) = 0$ (e.g. [29]), as shown in Fig. 2a. Nonetheless, the commonly utilised bisection search algorithm is not suitable for the SSRFEM method. From Fig. 2a and b, a significant difference can be observed when the bisection algorithm is applied to the two problems. That is, the evaluation cost for $f(x) > 0$ or $f(x) < 0$ is almost the same in the root finding problems, while in the SSRFEM method, the evaluation cost at the convergence side is remarkably smaller than that at the non-convergence side. Consequently, the

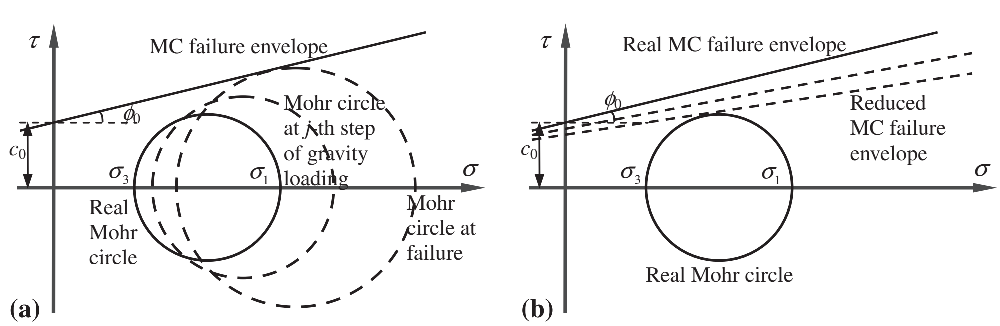

Fig. 1. Evolution of stress circle (a) incremental loading method; (b) shear strength reduction method.