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An approximation to the reliability of series geotechnical systems using a linearization approach

Peng Zeng *, Rafael Jimenez

ETSI Caminos, Canales y Puertos, Technical University of Madrid, Madrid 28040, Spain

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ABSTRACT

A method based on the linearization of the limit state functions (LSFs) is applied to evaluate the reliability of series geotechnical systems. The approach only needs information provided by first order reliability method (FORM) results: the vector of reliability indices, **β**, of the LSFs composing the system; and their correlation matrix, **R**. Two common geotechnical problems—the stability of a slope in layered soil and a circular tunnel in rock—are employed to demonstrate the simplicity, accuracy and efficiency of the suggested procedure, and advantages of the linearization approach with respect to alternative computational tools are discussed. It is also found that, if necessary, the second order reliability method (SORM)—that approximates the true LSF better than FORM—can be employed to compute better estimations of the system's reliability.

1. Introduction

Conventional geotechnical risk assessments mainly focus on deterministic or probabilistic analyses with individual failure modes (see e.g. [1–3]). During the past decades, system reliability has become a topic of intense research, covering aspects such as slope stability (see e.g. [4–11]), retaining walls [9,12] and tunnels [13].

Previous research on series geotechnical system reliability tends to focus on the use of (uni or bimodal) bounds to the probability of failure; several types of simulation-based approaches and variations of response surface methods (RSMs) have also been employed. But unimodal bounds [14] are often too wide to be useful and, although bimodal bounds [15] are narrower, they might be wide when the probabilities of failure corresponding to individual limit state functions (LSFs) are all 'large' (say, >0.01; see e.g. [16]), or when a large number of failure modes are considered [17]. In such cases, the bimodal bounds could be inappropriate.

Monte Carlo Simulation (MCS) [8], Importance Sampling (IS) [7,18] and the subset simulation method [19] offer unbiased estimators of the system's probability of failure, P_f. However, they could become unfeasible with computationally expensive problems (for instance, complex finite element models). Different types of RSMs—such as the classical RSM [20], ANN-based RSM [21], stratified response surfaces [22] and Kriging-based RSM [11,23]—have been proposed to partially overcome this drawback. However, they are approximate methods and, therefore, are not guaranteed to provide good estimators of P_f.

An alternative method, based on the linearization of the LSFs, is also possible to compute the reliability of series or parallel geotechnical systems [24–26]. However, for series systems, it has only been applied to very simple cases: e.g., short-term slope stability analyses of cohesive soils with two representative slip surfaces only [26], in which the (almost) linear nature of the LSFs makes it a natural solution. In other words, the ability of the linearization approach to perform well with other common series geotechnical problems is still untested. This note aims to illustrate the ability, in terms of simplicity, accuracy and efficiency, of the linearization approach to evaluate the reliability of two typical series geotechnical systems: a layered soil slope and a circular rock tunnel.

2. Approximation to the reliability of a series system

The probability of failure, P_f, of a series system can be approximated by transforming the random variables and LSFs to the independent standard normal space and linearizing the (transformed) LSFs at the design point. (The design point is the point in the failure domain closest to the origin of the independent standard normal space.) Based on the results (reliability indices and correlation matrix) of the first order reliability method (FORM), and following

* Corresponding author. Tel.: +34 633081799.
E-mail addresses: zengpeng1987@hotmail.com (P. Zeng), rafael.jimenez@upm.es (R. Jimenez).

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the convention that $g_i(\mathbf{X}) \leqslant 0$ indicates "failure", Cho [26] indicated that P_f can be computed through the unions and intersections of the failure domains associated with the hyperplanes tangent to each design point (see Fig. 1), as:

$$P_f = P\left(\bigcup_{i=1}^{n}\{g_i(\mathbf{X}) \leqslant 0\}\right) = \sum_{i=1}^{n} P_i - \sum_{i=1}^{n}\sum_{j>i}^{n} P_{ij} + \sum_{i=1}^{n}\sum_{j>i}^{n}\sum_{k>j}^{n} P_{ijk} - \cdots \quad (1)$$

$$P_{ij\cdots n} \approx \Phi_n(-\boldsymbol{\beta}; \mathbf{R}) \quad (2)$$

where n is the number of LSFs, $g_i(\mathbf{X})$ is the LSF in original (physical) space, $\Phi_n(-\boldsymbol{\beta}; \mathbf{R})$ is the cumulative density function (CDF) of the n-dimensional standard normal distribution evaluated for the vector of reliability indices, $-\boldsymbol{\beta} = [-\beta_1, -\beta_2, \ldots, -\beta_n]$, with correlation matrix, $\mathbf{R}$, given by $\mathbf{R}[i,j] = \boldsymbol{\alpha}_i^{\mathrm{T}}\boldsymbol{\alpha}_j$. ($\boldsymbol{\alpha}$ is the unit direction vector at the design point.) The probability of event-intersection given by Φ_n can be computed by the MATLAB pre-complied function **mvncdf**.

As will be demonstrated in Section 3.1, however, Eq. (1) could be cumbersome when the number of LSFs becomes large. (Note that $\sum_{i=2}^{n}\binom{n}{i}$ event-intersection reliability problems need to be solved.) In this context, building on Barranco-Cicilia et al. [27], Cho [26] suggested that "As an approximation, because the probabilities of event-intersections are generally small, terms higher than the second order in Eq. (1) can usually be neglected". But, as will be illustrated in Section 3, neglecting them may introduce significant errors when the probabilities of event-intersections higher than the second order are relatively large.

In this study, use is made of another strategy, originally due to Hohenbichler and Rackwitz [28], to overcome the aforementioned shortcomings and to solve series geotechnical systems simply, accurately and efficiently. For a series system with n LSFs, P_f can be computed through the complementary of the intersection of safe domains (see Fig. 1):

$$P_f = P\left(\bigcup_{i=1}^{n}\{g_i(\mathbf{X}) \leqslant 0\}\right) = P\left(\bigcup_{i=1}^{n}\{\bar{g}_i(\mathbf{U}) \leqslant 0\}\right) \approx P\left(\bigcup_{i=1}^{n}\{l_i(\mathbf{U}) \leqslant 0\}\right) = 1 - P\left(\bigcap_{i=1}^{n}\{-\boldsymbol{\alpha}_i\mathbf{U} < \beta_i\}\right) = 1 - \Phi_n(\boldsymbol{\beta}; \mathbf{R}), \quad (3)$$

where $\bar{g}_i(\mathbf{U})$ is the LSF in the (transformed) independent standard normal space and $l_i(\mathbf{U})$ is the linearization of $\bar{g}_i(\mathbf{U})$ at the design point. Note that Eq. (3) only needs to compute one event-intersection reliability problem. In general, the reliability index corresponding to the ith LSF can be computed using FORM, although as will be shown, the reliability indices computed with second order approximations (SORM) improve the computed estimates of P_f and might be needed when the LSFs are highly non-linear.

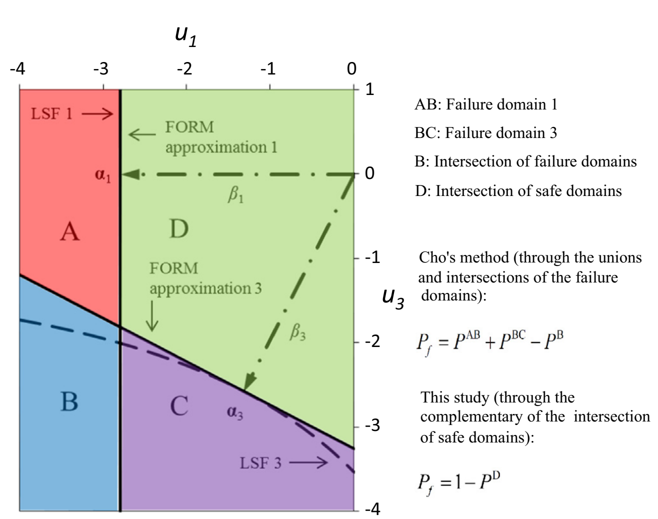

Fig. 1. A diagram in a two-dimensional standard normal space to illustrate the differences between Cho's method [26] and the method employed herein (*NOTE:* The LSFs shown correspond to β_1 and β_3 from Example 1.).

3. Case studies

Two typical geotechnical problems, taken from the literature, are here employed to illustrate the ability of the suggested method to compute the reliability of series geotechnical systems.

3.1. Soil slope in a layered profile

The probability of failure of a soil slope, where many slip surfaces are feasible, will be larger than for any individual slip surface; therefore, to compute the system's reliability, all potential slip surfaces would theoretically need to be considered [4]. Later research (see [10]), however, has shown that it is enough to consider a limited number of (weakly correlated) slip surfaces—those with a higher contribution to the system's probability of failure (for a discussion of how to identify them, see [26,29]).

To illustrate the ability of the linearization method to deal with a large number of LSFs, we start with the short-term analysis of the soil slope with 2 clay layers proposed by Ching et al. [7]; in particular, its reliability is computed using the 8 slip surfaces discussed by Low et al. [9]. Fig. 2 shows the geometry of the slope and the 8 slip surfaces considered; it also lists the means and standard deviations of two independent log-normal variables, C_{u1} and C_{u2}, employed to model the undrained shear strengths for both clay layers.

Table 1 lists the reliability index vector, $\boldsymbol{\beta}$, and the correlations between LSFs, $\mathbf{R}$, computed by Low et al. [9] using Bishop's simplified method with circular slip surfaces. (The LSFs are given by $G(\mathbf{X}) = F_S(\mathbf{X}) - 1$, where $\mathbf{X}$ is the vector of random shear strengths and F_S is the factor of safety of the slope.) Table 2 presents the result simulated with Monte Carlo by Ching et al. [7] for this slope—which can be considered as the "reference"—and compares it with bimodal bounds, the method suggested by Cho [26] and with the proposed linearization approach. Most of the methods considered provide very similar solutions, which are consistent with the MCS result; however, the method suggested by Cho [26] provides a P_f result that is far away from the 'exact' MCS result when terms higher than the second order are neglected—indeed, it is an infeasible solution, as it provides a negative P_f. The reason for such a difference is that neglecting terms higher than the second order could introduce significant errors for highly correlated LSFs.

On the other hand, the computational times for these methods are very different. The computational costs to evaluate P_f with Cho's [26] method when terms higher than the second order are neglected (0.04157 s) and the proposed linearization method (0.04877 s) are both minor and very similar (the computational times correspond to a MATLAB code run on a PC with an Intel Core i3-2100 CPU @3.10 GHz and 8 GB of RAM). But when terms higher