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Axisymmetric consolidation of a poroelastic soil layer with a compressible fluid constituent due to groundwater drawdown

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ABSTRACT

Axisymmetric consolidation of a poroelastic soil layer with a compressible fluid constituent induced by groundwater drawdown was studied based on Biot's axisymmetric consolidation theory. Laplace and Hankel transforms were employed to solve the governing equation. Explicit analytical solutions are obtained in the Laplace–Hankel transform domain when groundwater drawdown is induced by a constant pumping well. Based on the solutions, numerical computations were performed to study the influences of the compressibility of the fluid constituent on the consolidation behavior of the soil layer.

1. Introduction

Land subsidence is a worldwide problem that has caused substantial damage to infrastructure and resulted in great economic losses [1,2]. The problem is more severe in regions with soft soils where groundwater is heavily exploited for industrial and domestic purposes [3]. Two large regions in China that suffer from severe subsidence problems are the Yangtze River Delta and the North China Plain, where thick, soft soil layers exist [3–6]. In each of these regions, as groundwater is pumped from the aquifer, the water level and pore water pressure are reduced, which leads to an increase in effective stress and consolidation of the soil layer and thus results in land subsidence.

Because the permeability of an aquifer is usually much greater than that of a soil layer, the consolidation of a soil layer can be analyzed in two steps [7]. The first step is to compute or measure the groundwater drawdown in the aquifer. The second step is to compute the consolidation of the soil layer using the results of the first step as the boundary conditions. Several analytical solutions have been presented for the one-dimensional consolidation of a soil layer due to groundwater drawdown. Luo et al. [8] presented solutions for the one-dimensional consolidation of a soil layer due to constant groundwater drawdown. Li and Helm [9] presented solutions for the one-dimensional consolidation of a soil layer subjected to periodic groundwater fluctuation. Li [10] studied the one-dimensional consolidation of a nonlinear elastic soil layer subjected to periodic groundwater fluctuation. Teng et al. [11] studied the one-dimensional consolidation of a soil layer caused by constant groundwater drawdown with consideration of the body force effect. Tsai [12] studied the one-dimensional soil consolidation caused by constant groundwater drawdown with consideration of the viscosity effect. Liu et al. [13] presented solutions for the one-dimensional consolidation of a visco-elastic soil layer due to withdrawal of groundwater.

In one-dimensional consolidation theory, groundwater drawdown is assumed to be widely and uniformly distributed in an aquifer. However, because there are different types of groundwater drawdown that correspond to different pumping conditions used in practice, it is necessary to study the consolidation of soil layers based on Biot's three-dimensional consolidation theory [14,15]. On the other hand, the degree of saturation of a soil layer is usually not 100% because the soil may contain small bubbles. It has been reported that even a very small amount of gas in a soil dramatically increases the compressibility of the fluid [16]. Some researchers have studied the importance of fluid compressibility in the consolidation problem [16–20]. However, there have been no studies on the consolidation of a poroelastic soil layer with a compressible fluid constituent due to groundwater drawdown.

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Nomenclature

u_r	radial displacement
u_z	vertical displacement
r, θ, z	cylindrical coordinates
σ_{ij}	stress component
p	excess pore water pressure
ε_v	volumetric strain
ε_{ij}	strain component
G	shear modulus
k	coefficient of permeability
γ_w	unit weight of water
M	bulk modulus of pore fluid
v	Poisson's ratio
t	time variable
s	Laplace transform variable
ξ	Hankel transform variable
J_n	nth-order Bessel function of the first kind
a	conductivity of the aquifer
k_2	coefficient of permeability of the aquifer
Q	constant pumping rate
H	thickness of the soil layer
H_2	thickness of the aquifer

This paper presents a study of the axisymmetric consolidation of a poroelastic soil layer with a compressible fluid constituent due to groundwater drawdown based on Biot's axisymmetric consolidation theory. Laplace and Hankel transforms were employed to solve the governing equation. When groundwater is pumped from an aquifer at a constant rate, explicit analytical solutions are obtained in the Laplace–Hankel transform domain. Based on the solutions for groundwater drawdown induced by a constant pumping well, numerical computations were performed to study the influences of the compressibility of the fluid constituent on the consolidation behavior of the soil layer.

2. Mathematical model

2.1. Governing equations

It is assumed that the poroelastic layer is homogenous isotropic. The force equilibrium equations (with no body forces) specialized to axisymmetry are

$$\frac{\partial\sigma_{rr}}{\partial r}+\frac{\partial\sigma_{rz}}{\partial z}+\frac{\sigma_{rr}-\sigma_{\theta\theta}}{r}=0 \tag{1}$$

$$\frac{\partial\sigma_{rz}}{\partial r}+\frac{\partial\sigma_{zz}}{\partial z}+\frac{\sigma_{rz}}{r}=0 \tag{2}$$

where σ_{rr}, $\sigma_{\theta\theta}$, σ_{zz} are total normal stress components and σ_{rz} is shear stress component.

Assuming the solid constituent is incompressible, the constitutive equations take the form

$$\sigma_{ij}=2G\left(\varepsilon_{ij}+\frac{v}{1-2v}\varepsilon_v\delta_{ij}\right)-p\delta_{ij} \tag{3}$$

where the subscripts i and j can be r, θ or z, G is the shear modulus, v is the Poisson's ratio, p is the excess pore water pressure, δ_{ij} is the Kronecker delta.

The Navier-type equations for displacement and p are obtained by substituting Eq. (3) into Eqs. (1) and (2):

$$\left(\nabla^2-\frac{1}{r^2}\right)u_r+\frac{1}{1-2v}\frac{\partial\varepsilon_v}{\partial r}-\frac{1}{G}\frac{\partial p}{\partial r}=0 \tag{4}$$

$$\nabla^2u_z+\frac{1}{1-2v}\frac{\partial\varepsilon_v}{\partial z}-\frac{1}{G}\frac{\partial p}{\partial z}=0 \tag{5}$$

where $\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}$, u_r and u_z are the radial and vertical displacement components, respectively, $\varepsilon_v=\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z}$, is the volumetric strain.

It is assumed that the solid constituent is incompressible but the fluid constituent is compressible. Combined with Darcy's law, the pore fluid mass conservation equation is given by

$$\frac{k}{\gamma_w}\nabla^2p=\frac{\partial}{\partial t}\left(\varepsilon_v+\frac{p}{M}\right) \tag{6}$$

where k is the coefficient of permeability of the soil, γ_w is the unit weight of water, M is the bulk modulus (adjusted for porosity) of the pore fluid, and t is a time variable.

2.2. Boundary conditions

Fig. 1 illustrates a groundwater drawdown in an aquifer. The aquifer is assumed to be rough and rigid, so there is no vertical or radial displacement at the bottom boundary of the soil layer. The upper boundary of the soil layer is permeable and stress-free. The boundary conditions can be expressed as

$$z=0:\sigma_{zz}=0,\sigma_{rz}=0,p=0. \tag{7}$$

$$z=H:u_z=0,u_r=0,p=-\gamma_w h(r,t). \tag{8}$$

3. Solutions

3.1. General solutions

Laplace and Hankel transforms were employed to obtain the solutions. The nth-order Laplace–Hankel transform of $f(r, z, t)$ and its inversion may be found for example in Sneddon [21]:

$$\hat{f}(\xi,z,s)=\int_0^\infty\int_0^\infty f(r,z,t)rJ_n(\xi r)e^{-st}\mathrm{d}r\mathrm{d}t \tag{9}$$

$$f(r,z,t)=\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}\int_0^\infty\hat{f}(\xi,z,s)\xi J_n(\xi r)e^{st}\mathrm{d}\xi\mathrm{d}s \tag{10}$$

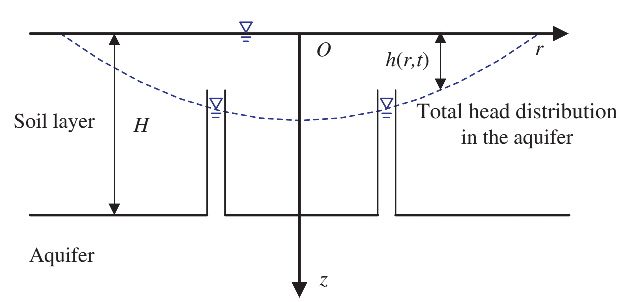

Fig. 1. Consolidation of a soil layer induced by groundwater drawdown in an aquifer.