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Global sensitivity analysis for subsoil parameter estimation in mechanized tunneling

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ABSTRACT

The present paper validates two alternative global sensitivity analysis methods, namely variance-based and elementary effect, for the purpose of detecting key subsoil parameters that influence the output of mechanized tunnel finite element simulation. In the elementary effect method, a strategy for considering the dependencies, that result from a set of constraints between different parameters, is proposed. Moreover, because the numerical implementation of variance-based sensitivity estimates, in particular, has been proven to require intensive evaluations of the system under investigation, a practical surrogate modeling technique is utilized. This technique is based on quadratic polynomial regression and represents a reliable approximation of the computationally expensive mechanized tunnel simulation. Furthermore, a convergence analysis based on Central Limit Theorem for the numerical implementation of the methods is introduced. The adopted analysis highlights model evaluations needed for the sensitivity measures to converge, as well as the uncertainty involved in these measures.

1. Introduction

There are different methods for performing sensitivity analysis SA; these methods are categorized into two main groups, global and local sensitivity analysis methods. In the local analysis, the partial derivatives of the system response with respect to its input variables are evaluated at a given base (local) point in the input space. Therefore, the information it gives about the sensitivities is totally dependent on the point at which the partial derivatives are evaluated, and hence, it is only suitable for linear models. On the other hand, the global methods explore the input parameter space, and therefore, the information it provides is independent of the model nature. However, the computational effort that it requires is notable in comparison to the local method. A complete review of sensitivity analysis techniques can be found in [1].

The sensitivity analysis techniques played an important role in modeling development in the last two decades. In this regard, due to the sophisticated mathematical models that have been developed to represent complex physical, engineering, social, or economical problems, the sensitivity analysis tools are of great importance for model calibration and for the determination of the key input parameters that governs the system responses. A recent review of applications of SA can be found in [1,2].

Sensitivity analysis techniques have been also employed in geotechnical applications, where, two recent employments can be found in [3,4]. Moreover, local sensitivity analysis for measuring the importance of subsoil parameters in parameter estimation and calibration studies of geotechnical boundary value problems has been utilized in [5–7]. However, according to the knowledge of the authors, global sensitivity methods, that have proven to be more efficient than local ones, have not been utilized for parameter estimation of subsoils so far.

In this paper we apply two global sensitivity analysis methods, namely *Variance-based* (VB) [8,1,9] and *elementary effect* (EE) [10,11], for the purpose of detecting the key subsoil parameters to be estimated in the process of system identification in mechanized tunneling. In this process, a computationally expensive numerical simulation of highly nonlinear nature of the geotechnical application with respect to both the physical and geometrical characteristics is carried out. Therefore, in order to make the identification process as efficient and robust as possible, it is favorable to determine the key subsoil parameters that influence the system response.

By the elementary effect method we attempt to identify the most important model parameters in relatively small number of evaluation points (simulation runs) that are well distributed in

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the parameter space. Whereas, in variance-based method a significantly large number of model evaluations is needed for the analysis, therefore, a practical surrogate model for the mechanized tunnel numerical simulation has been utilized.

The paper's outlines are as the following: Section 2 includes the adopted methods with an extension of the elementary effect method to consider constrained model parameters. Also, it includes the metamodel used to substitute the computationally expensive numerical simulation of mechanized tunnel with a concept of convergence analysis for the numerical implementation of the introduced global sensitivity analysis methods. In Section 3 the application of the global sensitivity analysis for ranking the subsoil parameters in mechanized tunnel simulation is showed. Section 4 summarizes the results.

2. Methodology

2.1. Variance-based method

A key point in sensitivity analysis is to study the model prediction uncertainty which results from its parameters uncertainty, therefore output variance analysis and study is one major objective. In this study the contribution of model parameters to the output variance is investigated and quantified. That is, the output variance is decomposed to the sum of contributions of each individual input parameter and the interactions (coupling terms) between different parameters.

A review of different VB methods is introduced in [12], and detailed information about the history of VB methods and their recent developments can be found in [1]. Based on the work of Sobol' [8], the VB sensitivity measures are represented as follows:

$$1 = \sum_i S_i + \sum_i \sum_{j>i} S_{ij} +, \cdots, +S_{12,\dots,k}, \tag{1}$$

in this equation, $S_i, S_{ij}, \dots, S_{12,\dots,k}$ are *Sobol's global sensitivity indices*. The *first order sensitivity index* S_i measures and quantifies the sensitivity of model response Y to the parameter X_i (without interaction terms), whereas, $S_{ij}, \dots, S_{12,\dots,k}$ are the sensitivity measures for the higher order terms (interaction terms).

Homma and Saltelli [13] have further proposed the *total effect* sensitivity index S_{Ti} which measures the whole effect of the variable X_i, i.e. the first order effect as well as its coupling terms with the other input variables.

The first order S_i and total effect S_{Ti} sensitivity indices are considered to be the major sensitivity indicators in VB methods, and they have been adopted in many real world applications [1] as well as in this work.

The Monte Carlo based numerical procedure for independent input variables proposed in [14] is adopted for estimating the sensitivity indices. In this procedure:

- Two independent (n, k) matrices $\mathbf{A}$ and $\mathbf{B}$, each contains n random samples of the input parameters vector $\mathbf{X} = X_1, X_2, \dots, X_k$, are generated.
- A third matrix $\mathbf{C}_i$ is defined, where all its columns are copied from matrix $\mathbf{B}$ except the ith column copied from its corresponding column in $\mathbf{A}$.
- The first order index is estimated based on [1] and the total effect index is calculated based on [15] as

$$S_i = \frac{\mathbf{y}_{\mathbf{A}}^T \mathbf{y}_{\mathbf{C}_i} - n(\bar{\mathbf{y}}_{\mathbf{A}})^2}{\mathbf{y}_{\mathbf{A}}^T \mathbf{y}_{\mathbf{A}} - n(\bar{\mathbf{y}}_{\mathbf{A}})^2}, \qquad S_{Ti} = \frac{(\mathbf{y}_{\mathbf{B}} - \mathbf{y}_{\mathbf{C}_i})^T (\mathbf{y}_{\mathbf{B}} - \mathbf{y}_{\mathbf{C}_i})}{2\mathbf{y}_{\mathbf{B}}^T \mathbf{y}_{\mathbf{B}} - 2n(\bar{\mathbf{y}}_{\mathbf{B}})^2}, \tag{2}$$

where $\mathbf{y}_{\mathbf{A}}, \mathbf{y}_{\mathbf{B}}$, and $\mathbf{y}_{\mathbf{C}_i}$ are vectors containing model evaluations for matrices $\mathbf{A}, \mathbf{B}$, and $\mathbf{C}_i$ respectively. $\bar{\mathbf{y}}_{\mathbf{A}}$ and $\bar{\mathbf{y}}_{\mathbf{B}}$ are the mean value estimates for the components of $\mathbf{y}_{\mathbf{A}}$ and $\mathbf{y}_{\mathbf{B}}$.

2.2. Elementary effect method

The concept of *elementary effect* has first been introduced by Morris in 1991 [10] with the aim of determining the importance of input variables to the system output. In this method, consider a model Y with k independent inputs $X_i, i = 1, \dots, k$, these inputs are normalized, and hence, the input space constructs a k-dimensional unit cube. This unit cube, is discretized into a p-level grid Ω. For a given point $\mathbf{X} = (X_1, X_2, \dots, X_k)$ in this grid, the elementary effect of the ith input parameter is defined as follows:

$$EE_i = \frac{Y(X_1, X_2, \dots, X_i + \Delta, \dots, X_k) - Y(X_1, X_2, \dots, X_k)}{\Delta}, \tag{3}$$

here, Δ is a value in $\{1/(p-1), \dots, 1 - 1/(p-1)\}$ with p being the number of levels. In calculating the elementary effect EE_i, the transformed point $(\mathbf{X} + \mathbf{e}_i\Delta), \mathbf{e}_i$ is a vector of zeros but with a unit as its ith component, is still in the grid Ω.

For a grid Ω with p-levels, the elementary effect EE_i associated with the factor i has a finite distribution F_i within Ω. The statistics of F_i, the mean μ and the standard deviation σ, are the sensitivity measures proposed by Morris. The mean μ estimates the overall influence of the input parameter to the system response, while the standard deviation σ detects the interaction effects with the other parameters as well as the nonlinear relation between the corresponding input and the system output. In other words, if σ is small, it implies that the elementary effect has small deviations from one point to another, and consequently, the influence of the interaction terms as well as the nonlinear relation between the system output and the corresponding input i is low. The opposite is true for large values of σ.

In the case of non-monotonic models or in the presence of interaction effects, the sign of the elementary effect might vary between different evaluation points. This change can reduce the value of the mean μ even for a factor that has a notable influence on the model output. Therefore, Campolongo et al. [11] proposed the use of μ^*, which is the mean of the distribution, G_i, of the elementary effect absolute value $|EE_i|$. In order to obtain as much as sensitivity information, Saltelli et al. [1] recommends computing the three statistics μ, σ, and μ^*, provided that, computing μ^* does not require any additional model evaluations.

2.2.1. Numerical estimation of sensitivity measures for independent input parameters

2.2.1.1. Morris grid. For the seek of generating an effective sample to estimate the statistics of the elementary effects, Morris [10] proposed an efficient experimental design for independent input variables. This design builds r trajectories of $(k+1)$ points within the discretized grid Ω. Each trajectory provides k elementary effects, one for each input factor, making the total number of sample points equal to $r(k+1)$. As an example, suppose a three dimensional input space $(k = 3)$ with a four-level gird $(p = 4)$ and Δ is chosen to be equal to $p/(2(p-1))$ i.e., $\Delta = 2/3$, a possible random trajectory is illustrated in Fig. 1(a). A detailed information about the sampling procedure with its optimization, which is presented by [11], can be found in [1].

After generating r trajectories, the sensitivity measures are computed as follows:

$$\mu_i = \frac{1}{r}\sum_{j=1}^{r} EE_i^j, \quad \sigma_i^2 = \frac{1}{1-r}\sum_{j=1}^{r}\left(EE_i^j - \mu_i\right)^2, \quad \mu_i^* = \frac{1}{r}\sum_{j=1}^{r}|EE_i^j|. \tag{4}$$

2.2.1.2. Random sampling. An alternative approach to Morris grid is to use Latin Hypercube (LH) sampling procedure [16] for generating a well distributed sample points in the input space and computing the elementary effects at each single point. In this