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On the use of effective stress in three-dimensional hydro-mechanical coupled model

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ABSTRACT

In the last decades, a number of hydro-mechanical elastoplastic constitutive models for unsaturated soils have been proposed. Those models couple the hydraulic and mechanical behaviour of unsaturated soils, and take into account the effects of the degree of saturation on the stress–strain behaviour and the effects of deformation on the soil–water characteristic response with a simple reversible part for the hysteresis. In addition, the influence of the suction on the stress–strain behaviour is considered. However, until now, few models predict the stress–strain and soil–water characteristic responses of unsaturated soils in a fully three-dimensional Finite Element code. This paper presents the predictions of an unsaturated soil model in a Three-dimensional Framework, and develops a study on the effect of partial saturation on the stability of shallow foundation resting on unsaturated silty soil. Qualitative predictions of the constitutive model show that incorporating a special formulation for the effective stress into an elastoplastic coupled hydro-mechanical model opens a full range of possibilities in modelling unsaturated soil behaviour.

1. Introduction

The advances in constitutive modelling of unsaturated soils in the last two decades have been numerous and wide-ranging and it is still an area of very active research at present [29,30]. Three different approaches are currently available to describe the behaviour of unsaturated soils: use of two stress state variables (net stress and suction), generalization of the effective stress concept, and mixture theories. All constitutive models require two independent stress measures as basic variables. Increasingly, the main stress variable includes saturation and suction. It is worth to say that the first two approaches are supported by thermodynamic considerations under the assumptions of continuous fluid phases.

The two stress variables, i.e. net stress ($\sigma_{ij} - p^a\delta_{ij}$) and matric suction s, have been successfully adopted in several developed models for unsaturated soils [12,27]. Alonso et al. [1] are the first to propose a comprehensive elastoplastic constitutive model for unsaturated soils using the two stress state variable approach. This model was developed within the framework of critical state soil mechanics and reduces to a Modified Cam Clay model [33] for the zero suction value. The major contribution of Alonso et al. [1] is the definition of a loading-collapse (LC) curve. This curve describes the variation of the preconsolidation stress with suction in a s-p plane, where p is the mean net stress ($\sigma_{ij}/3 - p^a$). Based on experimental results, Wheeler [39] suggested that Alonso et al.'s [1] model is incomplete in the sense that it provides no information on the variation of water content. Motivated by microscopic considerations, Wheeler [39] proposed a relationship to describe the variation of water content in deforming soils. This model, however, does not describe the capillary hysteresis and includes parameters that are difficult to determine.

In recent years, the modern development of constitutive models for unsaturated soils is tightly linked to the discussion on effective stress. Effective stress-based models [11,13,18,19,34,36,40] generally make use of Bishop's formula [10], i.e. $\sigma'_{ij} = \sigma_{ij} - p^a\delta_{ij} + \chi s\delta_{ij}$, where σ'_{ij} and σ_{ij} ($i,j = 1,2,3$) are components of effective stress and total stress tensors, respectively; δ_{ij} is the Kronecker delta; s is the matric suction ($s = p^a - p^w$); p^a and p^w are pore air and pore water pressures, respectively; and χ a general parameter weighting the contribution of each of the fluid phases in the distribution of the total stress.

It is recognized that effective stress approach provides a smooth transition from unsaturated to saturated states.

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In all models cited above, using the effective stress of Bishop, the parameter χ has been taken equal to the degree of saturation S_r. Furthermore, the variation of moisture content in the deforming soils is evaluated ad hoc by introducing the so-called soil water characteristic curves (SWCCs), i.e. a relationship between the moisture content (or degree of saturation) and the matric suction. Such a procedure is not sufficient to well describe the change of moisture content in an unsaturated soil [30]. In addition, capillary hysteresis phenomenon cannot be properly described in these models. Inclusion of hysteresis effects is essential to simulating the infiltration related problems in unsaturated soils [5,21,35]. Proper simulation of infiltration processes in unsaturated soils is important to analyzing the bearing capacity of unsaturated soils supporting a shallow foundation during or after heavy rainfall events and other complex phenomena. Wheeler et al. [40] presented an initial framework, within the two stress state approach, for coupling SWCCs and stress–strain behaviour. This model is only applicable to isotropic loading conditions and the assumed hysteresis in SWCCs is too simple to represent experimentally observed curves. The same representation for the SWCCs has been used by other authors [30]: the scanning curves (i.e. the variation of water content between the main drying and main wetting curves) are considered reversible. Recently more accurate models have been presented to describe the hysteresis in SWCCs (e.g. [23,31,38]). Experimental validations of these models are however, still limited, and the large number of parameters is prevalent. Several models have been developed to simulate SWCC's. In this field, we can cite also the work which led to the NAPL software [14]. This software is able to predict accurately the hysteresis effect on SWCC's, however the effect of the porous media deformation is not considered."

In this paper we first briefly review a hydro-mechanical elastoplastic model for unsaturated soils proposed by authors [3] then we present a new confrontation with experimental data. More precisely, we deal with the following:

(i) Description and evaluation of the proposed numerical hydromechanical coupled model for unsaturated soil focusing on: (1) the hydraulic model for SWCCs, to simulate correctly the hysteresis and porosity effects; (2) the definition for the coupling parameter χ; by comparing between experimental and numerical results for laboratory tests.
(ii) Implementation of the developed model in a Finite Element code Cast3M [37].
(iii) Evaluation of the effect of drying/wetting cycles on shallow foundation settlements, with a parametrical study for the effect of χ parameter.

The results from the studied Finite Element application show the relevance of the used definition for coupling parameters χ which leads to a full range of possibilities in modelling unsaturated soil behaviour and gives the effective stress of Bishop the ability to reproduce paradoxical phenomenon related to the effect of suction on the mechanical behaviour.

2. Elastoplastic model integrating the water retention behaviour

The elastoplastic constitutive model for unsaturated soils proposed by the authors [3] incorporates the influence of the degree of saturation on the stress–strain relations and strength, the influence of deformation on the soil–water characteristic response, and the influence of suction on both types of behaviour. This model is applicable to unsaturated soils in which the pore air and pore water are continuous throughout the voids. This paper adopts the model to simulate the hydraulic and mechanical behaviour of unsaturated soil in a three-dimensional Finite Element framework. In this section, the model is reviewed briefly.

2.1. Soil–water characteristic behaviour

An important feature of unsaturated soil behaviour is the irreversible change in volume and the saturation caused by cyclic drying/wetting under constant net stress. Due to hysteresis, the relationship between the saturation degree and suction takes several forms, depending on the incremental way to reach the suction value. So, the degree of saturation depends mainly on the wetting/drying history, suction and void ratio. There are four types of soil–water characteristic curves: (1) Boundary drying curve. (2) Boundary wetting curve. (3) Wetting scanning curves. (4) Drying scanning curves. An analytical expression has been used for each type of curves.

For the boundary curves, the Van Genuchten–Mualem equation has been used: two sets of empirical parameters (a_w, n_w) and (a_d, n_d) has been defined respectively for the boundary wetting and drying curves.

$$S_{rw} = S_{rres} + (S_{rsat} - S_{rres})\left[1 + \left(\frac{a_w s}{P_{atm}}\right)^{n_w}\right]^{\left(\frac{1}{n_w} - 1\right)}$$
$$S_{rd} = S_{rres} + (S_{rsat} - S_{rres})\left[1 + \left(\frac{a_d s}{P_{atm}}\right)^{n_d}\right]^{\left(\frac{1}{n_d} - 1\right)} \quad (1)$$

with S_{rres} and S_{rsat} being respectively the residual and full saturated degree of saturation. The atmospheric pressure P_{atm} is introduced to make the a_w and a_d parameters dimensionless.

The parameters n_w and n_d control the slope of the SWCCs when the suction values are greater than the Air Entry Value s_{AEV}.

s_{AEV} represents the suction limit where air starts to enter into the largest pores in the soil. The parameters a_w and a_d control numerically the s_{AEV}. In Eq. (2), the index p for (a_p; n_p; s_{AEV_p}) is identical to w in the wetting case and identical to d in the drying case.

$$a_p = \frac{P_{atm}}{s_{AEV_p}}\left(\frac{n_p - 1}{n_p}\right)^{\frac{1}{n_p}} \times \frac{n_p}{(1-n_p)(n_p-1)}\left(\left(\frac{2_{n_p-1}}{n_p}\right)^{\left(2-\frac{1}{n_p}\right)} - \left(\frac{2_{n_p-1}}{n_p}\right) + \frac{(1-n_p)(n_p-1)}{n_p}\right) \quad (2)$$

In order to take into account the effect of the porosity on the SWCCs, a modified exponential variation of the Air Entry Value s_{AEV} as function of porosity is introduced:

$$s_{AEV_p} = s_{AEV_{p0}} e^{\left(\lambda\left(\frac{1}{n} - \frac{1}{n_0}\right)\right)} \quad (3)$$

λ is a material parameter; n is the current porosity and $s_{AEV_{p0}}$ is a reference Air Entry Value for the porosity n_0.

A parametric study of the influence of parameter λ on the evolution of Air Entry Value is given in Fig. 1. It shows that the larger the value of λ, the smaller decrease in porosity is needed for varying the Air Entry Value, which leads to increase the effect of the porosity on SWCCs. To study the performance of the proposed model, the experimental data for drying water retention curves for compacted silty clay [32] under different initial void ratios, has been used (Fig. 2). The used parameters are summarized in Table 1.

Additionally, in order to capture the phenomenological behaviour of unsaturated soils, the non-linear and hysteretic characteristics of the water retention curve should be determined, including the so-called 'scanning curves'. A new approach inspired by the