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Analytical modeling of a deep tunnel inside a poro-viscoplastic rock mass accounting for different stages of its life cycle

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ABSTRACT

An analytical approach of the viscoplastic behavior of a porous saturated rock mass surrounding a deep tunnel in different stages of a simplified life cycle is presented. The viscoplasticity is modeled by means of a simple (linear) Norton-Hoff's law. Some numerical examples are performed to examine the consistency and relevance of the solutions. Parametric studies illustrate the influence of key parameters such as rock mass viscosity, Poisson's ratio, rock mass permeability, lining emplacement, etc. This analytical approach constitutes a useful reference to facilitate more complex numerical simulation and making it possible to obtain practical estimations of the poromechanical behavior of underground structures.

1. Introduction

This paper deals with hydromechanical behavior of a deep circular tunnel excavated in a poro-viscoplastic rock mass. It is a part of an on-going research project aiming at providing a clearer understanding of hydromechanical responses of underground structures following each stage of their service life.

The behavior of a deep tunnel inside an infinite, isotropic medium has often been addressed in the literature. The complexity of such models depends intrinsically on the constitutive behavior of the surrounding rock mass. In [1–3], we focused on an (quasi-) analytical approach of the post closure behavior of a cylindrical or spherical cavity drilled into a poroelastic or poroviscoelastic medium and submitted to a very simplified scenario (sudden application of the lithostatic stresses on the backfill after lining failure). A solution accounting for a more realistic (but still simplified) life cycle of the tunnel has been developed by Dufour et al. [4] in the particular case of poroelasticity. Inelastic strains of deep rocks under loading, which have been experimentally observed [5–7], have been taken into account in [8] by considering a poro-elastoplastic behavior of the rock mass. However, these authors focused their attention on the effects of hydraulic boundary conditions, while disregarding the effects of the transient evolution. Furthermore, most rocks, especially in the long-term, exhibit time-dependent irreversible strains as shown in [9,10]. Many authors used the theory of viscoplasticity to simulate this phenomenon, such as [11] or [12]. These last authors looked for a plausible explanation of the excess pore pressures observed in a borehole. Nonetheless, such a non-linear theory generally requires computational tools and do not easily lend itself to analytical approaches, which remain rare. Analytical approaches, however, are useful for quick order-of-magnitude estimates, in order to better understand the physical phenomena involved or to consolidate more sophisticated numerical models. Pouya [13] provided an approximate closed-form solution in the case of an unlined tunnel drilled into a dry viscoplastic rock mass obeying Norton-Hoff's creep law and is elastically incompressible ($\nu = 0.5$). Cosenza & Ghoreychi [14] have analytically dealt with the long-term behavior of a cavity inside a poro-viscoplastic rock mass but limited their attention to the special case of an asymptotic stationary state.

This paper proposes an analytical approach for the behavior of a deep tunnel inside a poro-viscoplastic rock mass, accounting for a work sequence idealized as 3 stages: (i) excavation; (ii) pore

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Nomenclature

$\mathbf{1}$	second order identity tensor
$\widehat{A}$	*translated* function of field A
$\dot{A}$	rate of a quantity A
A^*	normalized form of quantity$A$
$A!$	factorial of non-negative integer number A
a	radius of the tunnel
b	Biot's coefficient
E, E_L	young modulus of rock mass and lining support
e_L	thickness of lining support
$Int(x)$	integer part of x
I_0, K_0	modified Bessel functions of zero order of the first and second kind
K_1	modified Bessel function of the first order of the second kind
K_f	bulk modulus of fluid phase
K_L	stiffness of lining support
K_s	bulk modulus of solid phase of rock
K, G	bulk and Shear modulus of rocks in drained conditions
k_h	hydraulic diffusivity
$L\{f\} = \bar{f}$	laplace transform of function f
M	Biot's modulus
n	exponent of viscoplastic criterion
P_∞	initial geostatic pressure
p_{w0}	initial hydrostatic pore pressure
p_w	pore pressure
r	distance to the tunnel axis (space variable)
s	argument of Laplace transformed function
$\mathbf{s}$, $\mathbf{s}'$	deviatoric part of total and effective stress tensor
s_r, s_θ, s_z	components of deviatoric stress tensor
T_1, T_{hm}	characteristic time of creep and hydromechanical couplings
T_0, T_2	characteristic time of relaxation before and after lining emplacement
t_0, t_1, t_2	time of initial state, excavation and lining emplacement
t	time variable
$\mathbf{u}$	displacement vector
u	radial displacement
β	pore compressibility
$\boldsymbol{\varepsilon}$	strain tensor
$\boldsymbol{\varepsilon}^e$, $\boldsymbol{\varepsilon}^{vp}$	elastic and viscoplastic parts of the strain tensor
ε_v	volumetric strain
ϕ, ϕ_0	actual and initial porosity
ϕ^e, ϕ^{vp}	elastic and viscoplastic parts of the porosity
ϕ^*	dissipation potential
η	rock viscosity
λ_h	hydraulic conductivity
ν, ν_L	Poisson's ratio of rock mass and lining support
$\boldsymbol{\sigma}$, $\boldsymbol{\sigma}'$	total and effective stress tensor
σ_m, σ'_m	mean total and effective stress
σ_r, σ_θ, σ_z	components of total stress tensor
Ψ_s	free energy of the skeleton
Δ	laplace operator

pressure dissipation before lining emplacement; (iii) lining emplacement and tunnel operation. The rock mass is supposed to obey a simplified "linearized" ($n = 1$ in Eq. (9) below) Norton Hoff's law without hardening, nor creep-threshold. Note that according to Zhang [15], such assumptions seem to be acceptable for the Callovo-Oxfordian clay rock, considered as a suitable host-rock by ANDRA (The French National Radioactive Waste Management Agency). The presence of a saturating liquid phase is taken into account, using Coussy's theoretical framework [16] and a closed-form solution is obtained in the Laplace-Transformed domain. A quasi-analytical solution is obtained in the time domain using a numerical inversion. Finally, numerical applications are presented to show the consistency and pertinence of the proposed solution.

2. Simplified life cycle of a typical tunnel

The tunnel life cycle is idealized as 3 stages schematized in Fig. 1. Initially, at $t = t_0 = 0$, the medium is supposed to be in equilibrium with the hydrostatic pressure p_{w0} and the geostatic pressure P_∞ with zero displacements and strains. The first stage corresponds to the excavation of a circular tunnel of radius a in an infinite, isotropic and homogeneous poro-viscoplastic medium characterized by a Young's modulus E, a Poisson's ratio ν, a rock mass viscosity η and a rock hydraulic conductivity λ_h. The time required for excavation is supposed small compared to the characteristic creep time, so that excavation can be considered as instantaneous and ends at $t = t_1 = 0^+$. It is followed by the second stage during which the tunnel remains unlined and creep-induced convergence begins. The second stage ends when a lining support is emplaced at $t = t_2$. In the third stage, from $t = t_2$ onwards, the lining reaction at the tunnel wall slows down the creep-induced convergence. Note that a fourth stage (post-closure stage) could be envisaged from $t = t_3$ for backfilled tunnel but leads to complex mathematical developments and thus will not be considered hereafter.

3. General framework and resolution method of the problem

3.1. General framework of the problem

Plane-strain and cylindrical symmetry conditions are assumed so that the problem reduces to only two variables: r, the distance to the tunnel axisand t, the time. Isothermal conditions and small displacements and strains are also assumed; gravity is neglected, so that volumetric forces are not taken into account.

In the following, tensor and vector quantities are denoted in bold whereas scalars are in normal font. Under the above assumptions, the stress and strain fields $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ are diagonal, while the displacement field $\mathbf{u}$ is purely radial:

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_r & & \\ & \sigma_\theta & \\ & & \sigma_z \end{pmatrix}; \quad \mathbf{u} = \begin{pmatrix} u \\ 0 \\ 0 \end{pmatrix}; \quad \boldsymbol{\varepsilon} = \begin{pmatrix} \frac{\partial u}{\partial r} & & \\ & \frac{u}{r} & \\ & & 0 \end{pmatrix} \tag{1}$$

where $u = u(r, t)$ is the radial displacement. The volumetric strain ε_v is linked to the only non-zero component of the displacement vector as follows:

$$\varepsilon_v = tr(\boldsymbol{\varepsilon}) = \frac{\partial u}{\partial r} + \frac{u}{r} \tag{2}$$

From (1), the single non-trivial equilibrium equation from $\mathbf{div}(\boldsymbol{\sigma}) = 0$ is:

$$r\frac{\partial \sigma_r}{\partial r} + (\sigma_r - \sigma_\theta) = 0 \tag{3}$$

In order to represent the hydromechanical behavior of the rock, strain and porosity are considered as state variables as in [16]. The variation of porosity $(\phi - \phi_0)$ and the strain tensor $\boldsymbol{\varepsilon}$ are assumed to be decomposable into elastic and viscoplastic parts, which are respectively marked by superscripts "e" and "vp":

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^e + \boldsymbol{\varepsilon}^{vp}; \quad \phi - \phi_0 = \phi^e + \phi^{vp} \tag{4}$$