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The critical scale of fluctuation for active lateral forces in spatially variable undrained clays

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A R T I C L E I N F O

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A B S T R A C T

This study addresses the phenomenon of the critical scale of fluctuation (SOF) for active lateral force (P_a) in undrained clay when there is a spatial variability in the clay. The phenomenon is significant under shear strength (τ_f) random fields but is insignificant under unit weight (γ) random fields. It is found that the phenomenon of the critical SOF is connected to the nature of the spatial averaging, which is “line averaging” under τ_f random fields and is “area averaging” under γ random fields. The former averaging effect (line) is significantly weaker than the latter (area), so the tendency for the critical slip plane to seek for a favorable location is stronger for the τ_f random field than for the γ random field. Hence, the phenomenon of the critical SOF is more pronounced under τ_f random fields than under γ random fields. The underlying mechanisms for the phenomenon of the critical SOF will be explored in this paper.

1. Introduction

Spatial variability of soil properties has been extensively studied in recent years. More interestingly, it is shown that more complex behaviors can appear when the scale of fluctuation (SOF) is comparable to some multiple of the characteristic length of the structure [1] (e.g., height of slope, diameter of tunnel, depth of excavation). The complex behavior typically manifests itself as the discrepancy from the nominal behavior, e.g., the mean of the bearing capacity is less than the nominal value. In this study, the “critical SOF” or “worst case SOF” refers to the scale of fluctuation where the discrepancy becomes the largest. This critical SOF is observed in other studies as well [2–9]. Fenton and Griffiths [2] studied the bearing capacity of a strip footing on spatially random soils and indicated that there exists a worst case SOF that is approximately equal to the width of the footing. Fenton et al. [3] studied the retaining wall problems and found that the critical SOF is close to the wall height, at which the failure probability (p_f) becomes significantly larger. Breysse et al. [4] explored the soil–structure interaction issues and showed that there exists a critical SOF that is proportional to the characteristic dimension of the structure, at which the responses (such as differential settlements, moments, and stresses) are significantly larger. Griffiths et al. [5] and Soubra et al. [6] also illustrated the existence of the critical SOF for footings. More recently, the second author and his co-workers [7–9] found that the phenomenon of the critical SOF exists even in problems with very simple stress states.

Although the phenomenon of the critical SOF was observed in literature, its mechanism remains unclear. The objective of this study is to demonstrate one important aspect regarding the phenomenon of the critical SOF using the retaining wall example in undrained clays. In here, the critical SOF is defined as the scale of fluctuation at which the mean value of the active lateral force deviates the most from its nominal value. The phenomenon of the critical SOF is said to be significant if such deviation is large. It is found that there is a connection between the phenomenon of the critical SOF and the nature of the spatial averaging. The phenomenon will become significant when the spatial averaging effect is weak and will become insignificant when the effect is strong. The case with spatially variable shear strength (τ_f) *only* is taken to represent weak spatial averaging, as the spatial averaging only takes place along the potential slip plane (see Fig. 1) – it is “line averaging”. The case with spatially variable unit weight (γ) *only* is taken to represent strong spatial averaging, as the spatial averaging takes place over the entire triangular domain above the potential slip plane (Fig. 1) – it is “area averaging”. Note that unit weight is one of the least variable soil properties and is often treated as a deterministic variable in literature (e.g., [10]). Indeed, problems with spatially variable γ *only* are not interesting. We only use the case with spatially variable γ to exemplify strong spatial averaging, to contrast with the case with spatially variable τ_f. Such contrast will help to demonstrate the mechanisms for the phenomenon of the critical SOF. A case with spatially variable γ itself is not the focus of this study.

On the surface, our principle finding is that the phenomenon of the critical SOF is connected to the nature of the spatial averaging (namely, line or area averaging?). It is found that there are two

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Nomenclature

F	lateral force	μ_{τ_f}	inherent mean of τ_f
P_a	active lateral force	μ_γ	inherent mean of γ
$P_{a,n}$	nominal value for P_a	COV	coefficient of variation
p_f	failure probability	COV_{τ_f}	coefficient of variation of τ_f
FEA	finite element analysis	COV_γ	coefficient of variation of γ
RFEA	random field finite element analysis	SOF	scale of fluctuation
LEM	limit equilibrium method	δ_x	horizontal scale of fluctuation
H	wall height	δ_z	vertical scale of fluctuation
γ	unit weight	β	inclination angle of the potential slip plane
$\gamma(x,z)$	the random field for γ	ρ	auto-correlation
γ^{AA}	area average of $\gamma(x,z)$ over a potential slip wedge	SExp	single exponential model
s_u	undrained shear strength	Δx	horizontal distance
τ_f	shear strength	Δz	vertical distance
$\tau_f(x,z)$	the random field for τ_f	x	horizontal coordinate
τ_f^{LA}	line average of $\tau_f(x,z)$ along a potential slip plane	z	vertical coordinate
σ	inherent standard deviation	x^*	daylight position of the critical slip plane
μ	inherent mean		

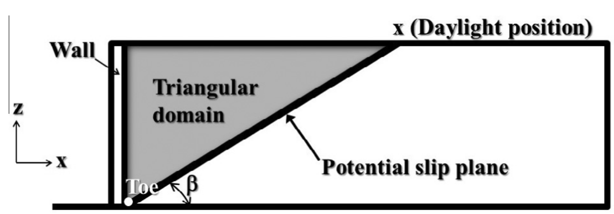

Fig. 1. Schematic for the potential slip plane (solid line) and the triangular domain (gray region) above the potential slip plane.

intermediate mechanisms between the phenomenon of the critical SOF and the nature of the spatial averaging. They are

1. The tendency for the critical slip plane to "seek" for a favorable location. For spatially variable τ_f, this tendency is to seek for the "weak path". For spatially variable γ, this tendency is to seek for the "heavy triangular domain".
2. The variability of the "averaged property function". This function describes how the line average of shear strength (or the area average of unit weight) varies with respect to the daylight position x (Fig. 1). This function is called the "averaged property function".

First, it is found that the phenomenon of the critical SOF is connected to the tendency of "seeking". Then, the tendency of seeking is, in turn, connected to the variability of the averaged property function. Finally, the variability of the averaged property function is connected to the nature of the spatial averaging. The aim of this paper is to demonstrate the numerical evidences for the connections among the aforementioned mechanisms.

2. Simulation of active lateral force P_a

Spatial variabilities of soil properties are commonly modeled by random fields [11]. In this study, an undrained clay is considered, i.e., the shear strength (τ_f) = the undrained shear strength (s_u). The shear strength (or unit weight) at a point is denoted by $\tau_f(x,z)$ (or $\gamma(x,z)$), where x and z are the horizontal and vertical coordinates, respectively. The soil property is simulated as a stationary Gaussian random field with inherent mean = μ and inherent standard deviation = σ. To define the correlation between two locations with horizontal distance = Δx and vertical distance = Δz, the single exponential (SExp) auto-correlation model is considered in this study [11,12]:

$$\rho(\Delta x, \Delta z) = \exp\left(-\frac{2|\Delta x|}{\delta_x} - \frac{2|\Delta z|}{\delta_z}\right) \quad (1)$$

where δ_x and δ_z are the horizontal and vertical SOFs. The two-dimensional (2D) stationary Gaussian random field can be simulated by the Fourier series method proposed in [13].

2.1. Simulating P_a under τ_f random field only

The limit equilibrium method (LEM) is adopted to simulate P_a samples under τ_f random field only (γ is a fixed constant). As mentioned previously, this represents the case with weak averaging effect, as spatial averaging only takes place along the potential slip plane (Fig. 1). The steps for this LEM have been presented in detail elsewhere [14]. The possible friction between the wall and soil is not considered. The idea is fairly simple. Numerous potential slip planes (all pass through the toe in Fig. 1) are produced. Each potential slip plane is characterized by the horizontal daylight position x (Fig. 1). The average of the τ_f values along each potential slip plane can be computed using the Fourier series method proposed by [13] (see Eq. (19) in [13]). This "line" average is denoted by $\tau_f^{LA}(x)$. Using force equilibrium, it is shown in [14] that

$$P_a = \max_x F(x) = \max_x \left(\frac{1}{2}\gamma H^2 - \tau_f^{LA}(x) \cdot \frac{x^2 + H^2}{x}\right) \quad (2)$$

where γ is the deterministic soil unit weight; H is the height of the retaining wall; $F(x)$ is the lateral force required to prevent the sliding of the triangular wedge above the potential slip plane (Fig. 1). Note that Eq. (2) does not consider the possible tension crack. In this case, P_a in Eq. (2) may include tensile stresses and can be negative if the soils can stand on its own.

The process of simulating P_a using LEM is illustrated in Fig. 2. Fig. 2a shows a realization of the τ_f random field, where x_A and x_B in Fig. 2a are the horizontal daylight positions of the two potential slip planes. There are actually infinite potential slip planes passing through the toe, but for clarity, only two of these planes are shown. Each potential slip plane has a line average τ_f^{LA}, and the resulting continuous line average process $\tau_f^{LA}(x)$ forms