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A constitutive model for unsaturated soils with consideration of inter-particle bonding

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ABSTRACT

The paper presents a physically-based constitutive model for unsaturated soils that considers the bonding effect of water menisci at inter-particle contacts. A bonding factor has been used to represent the magnitude of the equivalent bonding stress, defined as the bonding force per unit cross-sectional area. The average skeleton stress is employed to represent the effect of average fluid pressures within soil pores. Based on an empirical relationship between the bonding factor ζ and the ratio e/e_s (where e and e_s are void ratios at unsaturated and saturated states, respectively, at the same average skeleton stress), we propose an elasto-plastic constitutive model for isotropic stress states, and then extend this model to triaxial stress states within the framework of critical state soil mechanics. Because only one yield surface is needed in the proposed model, a relatively small number of parameters are required. Comparisons between experimental data and model results show that, in most cases, the proposed model can reasonably capture the important features of unsaturated soil behavior.

1. Introduction

Modeling the stress–strain behavior of unsaturated soils is one of the greatest challenges of geotechnical engineering. It has long been recognized that Terzaghi's effective stress can be used to successfully describe the mechanical behavior of saturated soils. But for soils under unsaturated conditions, the characterization of deformation behavior is difficult, owing to the existence of bonding forces induced by water menisci between soil particles. This bonding force acts at the points of contact between soil particles [1–3] and complicates the description of the soil mechanical behavior. This bonding force has a significant effect on the mechanical behavior of unsaturated soils and is closely related to degree of saturation, suction and pore size distribution. Some investigators have attempted to use a single stress variable to describe the stress–strain relationship of unsaturated soils [4–6] while others have used two independent constitutive variables [9,10,7,8]. Two independent constitutive variables are also used in the model presented in this paper.

In early elasto-plastic models, suction was commonly employed together with net stress to describe the mechanical behavior of unsaturated soils [11,1,12–14]. For example, Alonso et al. [11] assumed that normal compression lines are a function of suction and then derived the loading-collapse yield curve, which plays a central role in their model. The model is able to reproduce important features of unsaturated soils and has provided a basic framework for subsequent developments. For models of this kind, however, the capillary bonding effect of water menisci is measured by suction alone, which is not enough to properly describe real behavior.

Other elasto-plastic models have been proposed where the bonding effect is described by a constitutive variable which also includes degree of saturation [15–24]. Wheeler et al. [17] discussed the inter-relationships between the hydraulic and mechanical behavior in unsaturated soils emphasizing the role of the inter-particle forces associated to the water retention behavior. This bonding effect is also considered in a model by Buscarnera and Nova [23] that deals with mechanical instabilities in unsaturated soils. Recently, Zhou et al. [24] represented bonding and debonding effects in unsaturated soils by assuming that stiffness is a function of the effective degree of saturation.

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To the best of our knowledge, Gallipoli et al. [25] were the first to consider the effect of capillary bonding within an elasto-plastic constitutive model. They used the variable, ξ, to describe the magnitude of the bonding exerted by inter-particle water menisci:

$$\xi = f(s) \cdot (1 - S_r) \tag{1}$$

where S_r is water saturation and the term $(1 - S_r)$ accounts for the number of water menisci per unit volume of solid fraction; the function $f(s)$ expresses the ratio of the inter-particle attraction at the two suctions of s and zero for the ideal case of a water meniscus located at the contact between two identical spheres. This leads to a unique relationship between the bonding variable ξ and the ratio e/e_s, where e and e_s are void ratios calculated at the same value of average skeleton stress under unsaturated and saturated conditions, respectively [25,26]. The model is capable of reproducing many important features of unsaturated soil behavior with a single yield surface and requires only a small number of model parameters under isotropic stress conditions. The definition of the bonding variable in the model of Gallipoli et al. [25] is derived from physical considerations of a qualitative nature rather than from a closed-form calculation of the bonding force. This paper presents a different approach to the definition of the bonding variable based on the explicit consideration of the solid–liquid–gas geometry at the inter-particle contact in order to rigorously calculate the bonding stress.

This work assumes an idealized soil consisting of identical, regularly distributed, spherical particles. This means that all pores have the same shape and dimension, though this shape and dimension can change with changing void ratio. This implies that, for values of degree of saturation smaller than one, pore water exists only in the pendular regime and the presence of bulk water is neglected for the definition of the capillary bonding factor. This is of course a simplification because real soils have pores of different sizes and, for values of degree of saturation smaller than one, there will be smaller pores filled with water (bulk water) and larger pores filled with air, with water menisci at inter-particle contacts (meniscus water). In real soils, bulk water will only disappear at very low values of degree of saturation.

Despite these simplifying assumptions, which are necessary to obtain a closed form expression of the bonding force, the paper provides a useful theoretical framework for comparison against experimental data and allows a rigorous quantification of capillary bonding between soil particles in the pendular regime for different values of void ratio.

To rigorously represent the effect of capillary bonding, we define the bonding stress as the bonding force divided by the area that the force acts on. Then, we propose a relationship between the ratio (e/e_s) and the bonding stress, and validate that relationship against a series of experimental data from isotropic and triaxial loading tests. After that, we develop a physically based constitutive model, with a single yield surface, for isotropic stress states and then extend that model to triaxial stress states. A number of different experimental data sets are used to evaluate the model.

2. The bonding factor

For calculating the bonding force due to inter-particle water menisci, we conceptualize the soil as consisting of identical spherical particles. Consider a water meniscus between two such soil particles (Fig. 1). The relationship between the radius of curvature of the air–water interface r and the shortest distance from the contact point B to the interface r_1 can be described by

$$(\beta r + R)^2 = R^2 + (r + r_1)^2 \tag{2}$$

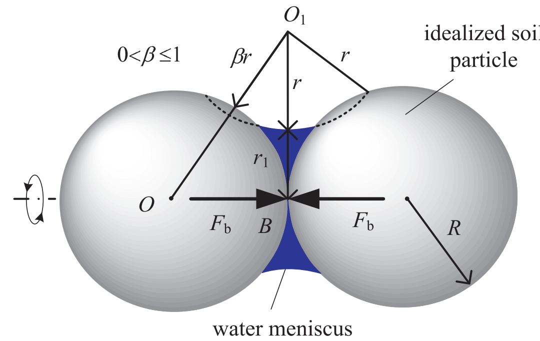

Fig. 1. Illustration of water meniscus between two identical contacting spheres.

where β is a parameter that decreases with increasing values of the contact angle ($\beta = 1$ for zero contact angle) and R is the radius of the idealized soil particles.

Unlike silt/sand/gravel particles, which have generally subrounded shapes, clay particles are flat and plate-like. Nevertheless, a clay compacted dry of optimum could still be represented as an assembly of aggregates of individual clay particles, where each aggregate is modeled as a sphere with an equivalent radius R [27] of the order of magnitude of 1 μm [25]. Along the same line, the clay soils are approximately represented by spheres in this study. In other words, the radius R in this study is referred as the average size of soil particles.

The capillary pressure, defined as the difference between pore air pressure and pore water pressure, is given by the Young–Laplace equation (e.g. [28]):

$$s \equiv p_a - p_w = T_s\left(\frac{1}{r} - \frac{1}{r_1}\right) \tag{3}$$

where p_a and p_w are the pore air pressure and pore water pressure (compression positive), respectively, and T_s is the surface tension of water. Note that $T_s = 0.0727$ N/m for a temperature of 20 °C [29] .

By neglecting gravity, the bonding force induced by the water meniscus in Fig. 1 can therefore be calculated as (e.g., [30,31])

$$F_b = \pi r_1^2 s + 2\pi r_1 T_s \tag{4}$$

Eq. (4) expresses the bonding force induced by water menisci. Note that there is another kind of the bonding force that is associated with clay mineralogy in the clayey soils, which does not closely relate to water menisci or the variation of saturation. Thus, we neglected the bonding stress associated with clay mineralogy in this study.

As previously indicated, our study is based on the assumptions that natural soils consist of identical soil particles approximated as spheres and the neglect of the bonding forces associated with clay mineralogy. Despite the limitations indicated above, Fig. 1 and the relevant equations provide a first approximation for soils, which leads to a simple and explicit expression of bonding stress. The reasonableness of our consideration was evaluated later by comparisons between model results and experimental data.

Substituting $\alpha = r_1/R$ into Eq. (2) yields

$$r = \begin{cases} \dfrac{\beta - \alpha - \sqrt{\alpha^2\beta^2 + \beta^2 - 2\alpha\beta}}{1 - \beta^2}R, & \text{when } \beta < 1 \\ \dfrac{\alpha^2}{2(1-\alpha)}R, & \text{when } \beta = 1 \end{cases} \tag{5}$$

Given that suction s must always be positive, α and β must meet the following condition: