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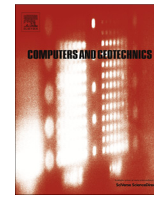

Implicit and explicit integration schemes in the anisotropic bounding surface plasticity model for cyclic behaviours of saturated clay

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A B S T R A C T

Two integration algorithms, namely the implicit return mapping and explicit sub-stepping schemes, are adopted in the anisotropic bounding surface plasticity model for cyclic behaviours of saturated clay and are implemented into finite element code. The model is a representative of a series of bounding surface models that have typical characteristics, including isotropic and kinematic hardening rules and a rotational bounding surface to capture complex but important cyclic behaviours of soils, such as cyclic shakedown and degradation. However, there is no explicit current yield surface in the model to which the conventional implicit algorithm returns the stress state back or the sub-stepping integration corrects the drift of the stress state. Hence, necessary modifications have been made for both of the integration schemes. First, the image stress point is mapped or corrected to the bounding surface instead of mapping back or correcting the stress state to the yield surface. Second, the unloading–loading criterion is checked to determine the image stress point rather than checking the yield criterion after giving the trial stress state in a conventional way. Comparative studies on the accuracy, stability and efficiency of the two integration schemes are conducted not only at the element level but also in solving boundary value problems of monotonic and cyclic bearing behaviours of rigid footings on saturated clay. For smaller strain increments, there is no significant difference in the accuracy between the two integration schemes, but the explicit integration shows a higher efficiency and accuracy. For relatively larger increments, the implicit return mapping algorithm presents good accuracy and more robustness, while the sub-stepping algorithm shows deteriorating accuracy and suffers the convergence problem. With the tolerance used in the present model, the bearing capacity of the rigid footing predicted by the return mapping algorithm is closer to the available analytical and numerical solutions, while the bearing capacity predicted by the sub-stepping algorithm shows a marginal increase.

1. Introduction

The response simulation of offshore structures embedded in seabed soils under cyclic loading still faces significant obstacles. First, it requires efficient and accurate constitutive models that reflect important cyclic behaviours of seabed soils, such as the hysteretic property, initial anisotropy, cyclic shakedown and stiffness degradation as well as the accompanying accumulation of plastic strain and pore pressure [1–3]. However, to capture all of these important but complex behaviours makes the constitutive model more lengthy and complicated. Moreover, in order to be applicable to offshore geotechnical calculations, the constitutive model requires efficient and robust numerical implementations, whereas the integration scheme of the incremental constitutive relations is the cornerstone that controls the accuracy, stability and efficiency of the calculations.

Existing approaches for stress integration of elasto-plastic constitutive models are generally classified as implicit and explicit schemes. Implicit algorithms that are based on the closest point projection or the return mapping [4–10] require a consistent tangent operator that corresponds to the final stress state of the integration increment. This arrangement means that an iterative calculation of the final stress state is needed. Explicit algorithms such as the algorithm with automatic error control and sub-stepping [11–14] require a continuum tangent operator that corresponds only to the initial stress state of the integration increment while using the adaptive sub-stepping to control the error. Both of the algorithms have been developed in classic elasto-plastic models but are still less reported for cyclic plasticity models. Manzari and Nour [7] first attempted to use an implicit algorithm in the bounding surface model for cyclic behaviours of soil. The results demonstrated the robustness of the implicit integration in

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the bounding surface model. However, one drawback of the model is the unrealistic description of cyclic loading because it is based on the fully isotropic hardening rule. Rouainia and Wood [8] presented an implicit return mapping integration in a modified bubble model based on a kinematic hardening rule, but it was only tested by a soil element. Borja et al. [9] used an implicit scheme to solve a two-surface model. However, the algorithm was run on the strain space in order to consider the nonlinear hyper-elasticity. Zhao et al. [15] argued that there were difficulties in the application of the implicit integration scheme to cyclic plasticity models and described the explicit integration of two complex constitutive models. However, they did not provide the performance of the algorithm in analysing the cyclic behaviour of the soil. Andrianopoulos et al. [16] proposed an explicit integration in the bounding surface model to analyse the earthquake liquefaction of noncohesive soils.

The accuracy, stability and efficiency of integration schemes are important issues in large-scale numerical simulation. However, comparative studies on the performance of the two integration algorithms in a complex cyclic plasticity model are rather limited. The conclusions from different researchers in solving boundary value problems are not uniform. Potts and Ganendra [17] compared the accuracy of return mapping implicit and sub-stepping explicit schemes in the Cam-clay model and stated that the sub-stepping algorithm was more accurate for a specific incremental size and for the analysis of a cavity expansion problem. Manzari and Prachathananukit [18] compared the closest point projection implicit integration with the sub-stepping explicit integration in a two-surface model and implemented them into finite element code. It was observed that for a relatively large strain increment, the implicit algorithm remained stable and accurate, while the explicit algorithm faced convergence difficulties. Sołowski et al. [19] ran both implicit and sub-stepping explicit integrations in the Barcelona basic model of unsaturated soil at a single stress point. However, it was concluded that for a larger strain increment, the implicit scheme offered faster convergence but might cause inaccurate computations. These findings highlight the importance of comparative studies on the accuracy, stability and efficiency of the two integration schemes.

The bounding surface plasticity model with a vanishing elastic region is more attractive for large-scale mathematical modelling related to cyclic loading because it is not necessary to address the evolvement of more than two yield surfaces (such as in the two-surface and multi-surface plasticity models [20–22]) and the smooth translation from nonlinear elastic to elasto-plastic behaviours. A recently developed anisotropic bounding surface model [23] has been shown to realistically present the stress–strain behaviours of the soils, including the cyclic shakedown and degradation. The present work is to implement the developed model with a vanishing elastic region [23] into a commercial finite element code with two integration schemes, i.e., the return mapping and sub-stepping integration schemes. However, there is no explicit current yield surface in the model to which the conventional implicit algorithm returns the stress state back or the sub-stepping integration corrects the drift of the stress state. Several necessary modifications should be made for both of the integration schemes. The performance, including the accuracy, robustness and efficiency of the two integration schemes, is investigated in detail both at the element level and in solving boundary value problems that involve monotonic and cyclic bearing behaviours of rigid footings on normally consolidated saturated clay.

2. Outline of the anisotropic bounding surface model

In this section, the anisotropic bounding surface plasticity model with a vanishing elastic region for saturated clay proposed by Hu et al. [23] is generalised to the multiaxial stress space. Within the framework of critical state soil mechanics, this model has been shown to accurately simulate important characteristics of saturated clay under cyclic loading such as initial anisotropy, reversal flow, cyclic shakedown and stiffness degradation by combining isotropic with kinematic hardening rules and adopting a rotational bounding surface. A brief description of the model is presented below.

In terms of notation, tensors are written in bold face characters to allow them to be easily distinguished from scalars. All of the presented stress quantities are effective. The symbol ':' denotes an inner product of two second-order tensors (e.g., $\mathbf{c}:\mathbf{d} = c_{ij}d_{ij}$) or a double contraction of the adjacent indices of tensors of rank two and higher (e.g., $\mathbf{C}:\boldsymbol{\varepsilon}^e = C_{ijkl}\varepsilon^e_{kl}$). The symbol '$\otimes$' denotes the Kronecker product of two second-order tensors (e.g., $\mathbf{c}\otimes\mathbf{d} = c_{ij}d_{kl}$).

2.1. Bounding surface formulation

For the initial consolidation process, the form of the bounding surface in the model proposed by Hu et al. [23] is the same as the form adopted by Dafalias [24], which can be written in the conventional triaxial p–q stress space as

$$F = \bar{p}^2 - \bar{p}p_c + \frac{(\bar{q} - \alpha\bar{p})^2}{M^2 - \alpha^2} = 0 \tag{1}$$

where $\bar{p}$ and $\bar{q}$ are mean effective and deviatoric stresses, respectively, and the superimposed bar indicates that the variables are related to the bounding surface; M is the slope of the critical state line and equals M_e for extension and M_c for compression; p_c and α define the size and inclination of the bounding surface, respectively, and their initial values are denoted by p_0 and α_0. The concept of the model is shown graphically in Fig. 1 in the p–q stress space.

The generalisation of Eq. (1) in the multiaxial stress space is obtained by standard methods [25,26], as follows:

$$F = \bar{p}^2 - \bar{p}p_c + \frac{3}{2(M^2 - \alpha^2)}[(\bar{\mathbf{s}} - \bar{p}\boldsymbol{\alpha}) : (\bar{\mathbf{s}} - \bar{p}\boldsymbol{\alpha})] = 0 \tag{2}$$

where $\bar{\mathbf{s}}$ and $\boldsymbol{\alpha}$ are deviatoric and anisotropic tensors, respectively, and $\alpha = \sqrt{\frac{3}{2}\boldsymbol{\alpha}:\boldsymbol{\alpha}}$ is a measure of the degree of soil anisotropy.

It can be seen from Eq. (2) that the bounding surface passes through the origin of the stress space. However, for the sequence shearing after the initial consolidation process, the model [23] has assumed that the bounding surface translates according to the kinematic hardening rule, which will be briefly explained in the following section (the details can be found in Ref. [23]). As a result, the endpoint of the bounding surface, which coincides with the origin of the stress space in the initial consolidation process, will translate to a new position in the stress space. We denote the endpoint as ξ (Fig. 1). Hence, the translating bounding surface in the multiaxial stress space is expressed as

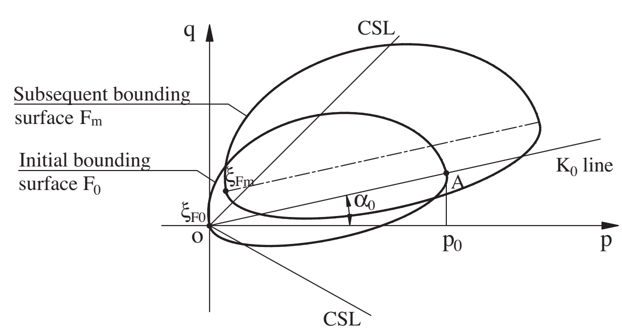

Fig. 1. Schematic of the rotational bounding surface in the p–q space.