Computers and Geotechnics 55 (2014) 57–66

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Applying bi-directional evolutionary structural optimisation method for tunnel reinforcement design considering nonlinear material behaviour

T. Nguyen *, K. Ghabraie, T. Tran-Cong

Computational Engineering and Science Research Centre (CESRC), Faculty of Engineering and Surveying (FoES), University of Southern Queensland (USQ), Toowoomba, QLD 4350, Australia

ARTICLE INFO

Article history:
Received 15 April 2013
Received in revised form 30 July 2013
Accepted 31 July 2013
Available online 27 August 2013

Keywords:
Topology optimisation
BESO method
Nonlinear material behaviour
Tunnel reinforcement

ABSTRACT

In the empirical methods for reinforcement design of underground excavations, an even distribution of rock bolts is generally recommended. This work proves that this design is not necessarily optimal and shows how the state-of-the-art reinforcement design could be improved through topology optimisation techniques. The Bidirectional Evolutionary Structural Optimisation (BESO) method has been extended to consider nonlinear material behaviour. An elastic perfectly-plastic Mohr–Coulomb model is utilised for both original rock and reinforced rock. External work along the tunnel wall is considered as the objective function. Various *in situ* stress conditions with different horizontal stress ratios and different geostatic stress magnitudes are investigated through several examples. The outcomes show that the proposed approach is capable of improving tunnel reinforcement design. Also, significant difference in optimal reinforcement distribution for the cases of linear and nonlinear analysis results proves the importance of the influence of realistic nonlinear material properties on the final outcome.

1. Introduction

The essential function of tunnel reinforcement system such as rock bolting is to strengthen the tunnel by integrating and transferring a major portion of load to the host ground [14]. Design of bolting of rock mass can be viewed as finding the reinforcement distribution in the host ground. Currently, tunnel reinforcement design mainly relies on past experience or empirical recommendations based on ground classification system. Topology optimisation techniques which are capable of finding the optimal material distribution are also applicable to solve reinforcement design problems in rocks. In spite of robust applications of topology optimisation in a wide range of engineering areas, there have been limited works on applying these design optimisation techniques in geotechnical engineering and particularly in tunnelling engineering.

Topology optimisation is to search for the optimal material distribution in the assigned design domain. A huge number of published works has been devoted to this interesting topic throughout the last decades. Some of the most popular optimisation methods developed can be listed as homogenisation method, solid isotropic material with penalisation (SIMP) method, evolutionary structural optimisation (ESO) method and bi-directional evolutionary structural optimisation (BESO) method. In the homogenisation technique proposed by Bendsoe and Kikuchi [2], artificial microstructure composites composed of voids and solids are considered as the building blocks of the structure at the microscopic level. The macroscopic properties of the structure was then calculated by homogenisation theory and the optimisation problem is solved based on optimality criteria. This method is effective for elastic problem and normally requires a considerable number of design variables [8]. SIMP method defines material constitutive tensor at each element as a function of material density varying from very small value (representing voids or weak elements) to 1 (representing solids or strong elements) [1,29]. A penalty factor is employed to penalise the intermediate density value and consequently push the elements to the two extremes of solid and void. However, the obtained topology is dependent on penalty factor and some grey elements are observed in the final results even if a large penalty factor value is utilised [20].

Xie and Steven [23] presented ESO method with a fairly simple concept of gradually removing inefficient elements from the structure towards its optimal shape and topology. BESO method is an extension of ESO method by allowing efficient elements to be added as well [18,25]. This method is easy to implement as it can be effectively linked with all finite element analysis packages.

* Corresponding author. Tel.: +61 449782892.
E-mail addresses: Tin.Nguyen@usq.edu.au, ngtin.gte@gmail.com (T. Nguyen), Kazem.Ghabraie@usq.edu.au (K. Ghabraie), thanh.tran-cong@usq.edu.au (T. Tran-Cong).

0266-352X/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.compgeo.2013.07.015

Also, the final results has no grey elements and hence much clearer than the SIMP method.

Initially, the above-mentioned methods are designed for optimisation problems for linear material behaviour. Effects of nonlinear material behaviour on topology optimisation design have been studied later in some works [17,4,13,10]. However, consideration of topology optimisation for multiple nonlinear material working together has not been fully studied. This work is going to improve and extend the BESO method for two types of nonlinear material and apply it into tunnel reinforcement design. It will be shown that the derived sensitivities could be used with any material model.

A small number of studies has been focused on extending topology optimisation techniques to tunnel design area. Yin et al. [28] initiated by applying the homogenisation method in which every element in the design domain is assumed as a square cell made of original rock surrounded by reinforced rock. The external work along the tunnel wall has been minimised under a prescribed reinforcement volume. Yin and Yang [26] conducted further research on optimising tunnel support in various layered geological structure conditions. The SIMP method was employed in their work with a power-law interpolation to determine the optimum distribution of reinforcement density in the design domain. Another severe issue in tunnel reinforcement design, namely tunnel and sidewall heave caused by swelling or squeezing rock, was addressed by Yin and Yang [27]. Liu et al. [16] tackled a similar problem by a different approach, namely Fixed-Grid Bidirectional Evolutionary Structural Optimisation (FG BESO). This approach was used to overcome the mesh-dependency and zigzag boundary problem in intermediate and final results. With regards to the shape optimisation of underground excavations, Ren et al. [19] and Ghabraie et al. [6] demonstrated the ability of ESO method in searching for the optimal shape based on stress distribution. A simultaneous optimisation of shape and distributed reinforcement of an underground excavation for elastic material was also explored by Ghabraie et al. [7] using BESO method.

These previous researches merely considered homogeneous, linear elastic material behaviour. However, linear character is not always a valid assumption especially for geomechanics materials and should only be considered as the initial step to model geotechnical problems [12]. Therefore, topology optimisation for nonlinear geomaterial is a reasonable extension from the previous studies in order to obtain proper and more reliable designs.

In this paper, the BESO method is improved and extended to handle nonlinear materials in the search for an optimal reinforcement distribution in underground excavation design. In the following sections, firstly, modelling techniques for tunnel excavation and reinforcement are demonstrated. A nonlinear sensitivity analysis is derived based on the adjoint method, followed by some specific numerical implementations. Procedures for swapping two types of considered material (original rock and reinforced rock) are stated. Some examples and discussions are presented to verify and illustrate the effectiveness of the proposed approach.

2. Tunnel modelling

Inhomogeneous, anisotropic and nonlinear properties are natural characters of geomechanical materials [12]. However, capturing all of these factors simultaneously in a topology optimisation is not an easy task. In the previous studies on the application of topology optimisation in tunnel reinforcement design, extremely simplified isotropic, homogeneous, linear elastic material models were adopted. This study extends these previous works by considering non-linear material models. The authors are currently working on removing the other limiting assumptions.

Two types of materials are concerned, i.e. original rock and reinforced rock. Mohr–Coulomb constitutive model which has been broadly used in geotechnical engineering applications due to its simplicity and acceptable accuracy is employed to model original and reinforced rock. It should be emphasised however, that the approach proposed here is not dependent on the material model used. The material parameters are shown in Table 1.

The tunnel is assumed to be long and straight enough to satisfy plane strain assumption. Fig. 1 shows the initial guess reinforcement design which is based on a current empirical recommendation. In this figure, the reinforced zone is illustrated by dark grey area. Elements around the tunnel opening are assigned as non-designable elements, denoted by black coloured elements and assumed to have the same properties as reinforced material. Owing to symmetry, only half of the design domain is modelled in the finite element analysis with proper symmetric conditions. Excavation process is analysed by finite element method using ABAQUS (version 6.11). Since the case of deep tunnel is investigated, the effect of difference in gravity force is ignored here. The geometry of the tunnel is a rectangle of size $w \times h = 10\text{ m} \times 5\text{ m}$ augmented at the top with a semi-circle of radius 5 m. After some numerical testings, the design domain geometry is selected as a square of side length 8 h. This is to ensure that the design domain size does not have a noticeable effect on topology optimisation results.

Unlike linear material models, loading history affects the response of nonlinear materials. Thus, the tunnel excavation is modelled in two separate steps. Firstly, pre-excavation conditions are obtained by restraining the nodal displacements of elements at the tunnel surface and at the boundary regions between the reinforced rock and the original rock. In the subsequent step, the displacement restraints along the boundary of the opening are removed.

3. Objective function and nonlinear sensitivity analysis

The aim of the tunnel reinforcement design is to employ a minimum amount of reinforcements while tunnel deformation needs to be limited. In the optimisation perspective, this objective can be stated as finding the minimum tunnel deformation under a prescribed reinforcement volume. As seen in the proposed method below, the optimisation process minimises the external work along the tunnel wall, which measures the tunnel face deformation, under a constrained reinforcement volume. The outcomes provide an optimal rock bolt distribution for a certain amount of bolt volume in order to obtain a minimum external work. The optimisation problem can be stated as

$$\begin{aligned} \min W = & \int \mathbf{f}^T d\mathbf{u} = \lim_{n\to\infty}\left[\frac{1}{2}\sum_{i=1}^{n}(\mathbf{u}_i^T - \mathbf{u}_{i-1}^T)(\mathbf{f}_i + \mathbf{f}_{i-1})\right] \\ \text{subject to}: \quad V_R = & \sum_{e=1}^{M} V_e x_e \\ x_e \in & \{0,1\} \end{aligned} \tag{1}$$

where W is the total external work, $\mathbf{u}$ the displacement vector, $\mathbf{f}$ the external force vector as shown in Fig. 1, n the number of iterations in solving the non-linear equilibrium equations, V_e the volume of

Table 1
Engineering properties of original rock and reinforced rock.

Material properties	Original rock	Reinforced rock
Young modulus (GPa)	0.1	0.5
Poisson's ratio	0.3	0.3
Friction angle (°)	27	32
Cohesion (MPa)	0.1	0.3