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Large deformation dynamic analysis of saturated porous media with applications to penetration problems

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ABSTRACT

This paper outlines the development as well as implementation of a numerical procedure for coupled finite element analysis of dynamic problems in geomechanics, particularly those involving large deformations and soil-structure interaction. The procedure is based on Biot's theory for the dynamic behaviour of saturated porous media. The nonlinear behaviour of the solid phase of the soil is represented by either the Mohr Coulomb or Modified Cam Clay material model. The interface between soil and structure is modelled by the so-called node-to-segment contact method. The contact algorithm uses a penalty approach to enforce constraints and to prevent rigid body interpenetration. Moreover, the contact algorithm utilises a smooth discretisation of the contact surfaces to decrease numerical oscillations. An Arbitrary Lagrangian–Eulerian (ALE) scheme preserves the quality and topology of the finite element mesh throughout the numerical simulation. The generalised-α method is used to integrate the governing equations of motion in the time domain. Some aspects of the numerical procedure are validated by solving two benchmark problems. Subsequently, dynamic soil behaviour including the development of excess pore-water pressure due to the fast installation of a single pile and the penetration of a free falling torpedo anchor are studied. The numerical results indicate the robustness and applicability of the proposed method. Typical distributions of the predicted excess pore-water pressures generated due to the dynamic penetration of an object into a saturated soil are presented, revealing higher magnitudes of pore pressure at the face of the penetrometer and lower values along the shaft. A smooth discretisation of the contact interface between soil and structure is found to be a crucial factor to avoid severe oscillations in the predicted dynamic response of the soil.

1. Introduction

Numerical modelling of the dynamic loading of saturated soil bodies undergoing large displacements and possibly surface penetration is one of the most sophisticated and challenging problems in computational geomechanics, mainly due to the extreme material and geometrical nonlinearity, large distortions, changing boundary conditions, material rate effects, and the inertia forces induced in the soil. Moreover, the presence of a pore fluid in a saturated soil and the generation of excess pore fluid pressures due to the dynamic loading, together with the subsequent consolidation arising from the dissipation of those excess pressures, combine to increase the analytical complexity of such problems. A fully coupled analysis is required in order to capture all aspects of the dynamics of the saturated soil behaviour. The analysis of a free falling penetrometer (FFP) is a particular example of one of these geotechnical problems where dynamic effects cannot be neglected. Numerical solution schemes for such problems require robust algorithms for time stepping, domain remeshing, interface modelling and stress–strain integration of the soil constitutive model.

When a saturated soil is loaded under undrained conditions, excess pore-water pressures are usually developed. The majority of current numerical models for simulating objects penetrating into soils are generally based upon a displacement formulation involving a single-phase soil, where the excess pore water pressures are not explicitly calculated. These methods, assuming saturated conditions, can only predict the total stresses developed in soil and in general they cannot be used to separately find the excess pore water pressures and the effective stresses in soil. Moreover, the effect of inertia forces on excess pore pressure generation is not taken into account.

This paper presents a computational framework for the analysis of the dynamic loading of a saturated soil body undergoing large displacements including surface penetration. The method is based on the Arbitrary Lagrangian Eulerian approach developed for dynamic and consolidation analysis of geotechnical problems [1,2], and the generalised-α time-integration scheme extended to dynamic consolidation problems [3]. In particular, this paper

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investigates the penetration of objects into a nonlinear saturated porous medium, and addresses the pore water pressure distribution as well as the soil resistance predicted during the installation phase. The soil is modelled as water-saturated porous finite elements and the penetrating object is discretised using impermeable and rigid finite elements. The interface between the soil and the penetrometer is modelled using the so-called node-to-segment contact method. Two material models, including the Mohr–Coulomb (MC) and the Modified Cam Clay (MCC) models, are used to represent the nonlinear behaviour of the soil. An Arbitrary Lagrangian Eulerian (ALE) [1,2,4] scheme is employed to preserve the mesh quality and topology throughout the numerical simulation and to avoid excessive mesh distortion. In the following, the numerical method including the governing equations and solution procedures are presented. Two numerical examples are then solved to validate important aspects of the numerical algorithm. Finally, a few examples of dynamic penetration are presented to illustrate the robustness and performance capabilities of the computational method.

2. Governing equations

The solution of coupled problems requires that the soil and pore fluid interaction should be analysed by developing a multiphase continuum formulation in order to incorporate the effect of the transient flow of the pore fluid through the interconnected voids of the solid soil skeleton. Biot [5] presented one of the first theories governing the behaviour of saturated porous media. A great deal of research has been devoted to studying his dynamic poro-elasticity theory, leading to various numerical and analytical solution procedures. Based on Biot's model and the finite element method, Ghaboussi and Wilson [6] proposed a numerical approach for solving coupled problems of geomechanics. Later, Small [7] and Carter et al. [8] extended the method for solving problems involving material and geometrical nonlinearity, respectively. Zienkiewicz and Shiomi [9] conducted a comprehensive study on the solution of Biot-type formulations and identified three alternative methods of analysis, including the so-called **u**–**p**, **u**–**U** and **u**–**p**–**U** formulations, in which solid displacements, **u**, pore fluid pressure, **p**, and pore fluid displacements, **U**, are the field quantities. In order to simplify the solution process, the acceleration of the fluid component of the soil can usually be ignored, as this term is generally not significant compared to the motion of the soil skeleton [9]. Ignoring the fluid acceleration results in a coupled set of equations in which **u** and **p** are the only unknowns. This type of formulation is termed "**u**–**p**", and is adopted in this study. In the **u**–**p** formulation, the momentum balance of the soil–fluid mixture states that

$$\sigma_{ji,j} - \rho \ddot{u}_i + \rho b_i = 0 \tag{1}$$

where σ is the total stress, b represents the body force, ρ is the density of the fully saturated porous medium, and u denotes the acceleration of the solid skeleton. According to the principle of effective stress, the total stress is equal to the sum of the effective stresses, σ', and the pore-water pressure, p, i.e.,

$$\sigma_{ij} = \sigma'_{ij} + \delta_{ij} p \tag{2}$$

where δ_{ij} is the Kronecker delta.

Applying the standard finite element procedure, the matrix form of the momentum balance equation is obtained:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} + \mathbf{L}\mathbf{p} = \mathbf{f}^u \tag{3}$$

where **M**, **C**, **K**, and **L** are, respectively, the mass, damping, stiffness, and coupling matrices, a superimposed dot represents the time derivative of a variable and $\mathbf{f}^u$ is the vector of external nodal forces. Details of these matrices and vectors are given in Appendix A.

In a coupled **u**–**p** analysis, the momentum balance of the pore fluid–solid mixture is fulfilled by

$$p_{,i} + \rho_f b_i - k_{ij}^{-1} w_j - \rho_f \ddot{u}_i = 0 \tag{4}$$

and the conservation of mass is satisfied by

$$w_{i,i} + \dot{\varepsilon}_{ii} + \frac{\dot{p}}{Q} + \dot{S}_0 = 0 \tag{5}$$

where w is the average (Darcy) velocity of the pore fluid seepage, k is the dynamic permeability, ρ_f is the density of the pore fluid, ε is the strain, Q is the bulk stiffness of the mixture, and $\dot{S}_0$ represents the fluid supply. Note that the term $\frac{\dot{p}}{Q}$ accounts for the additional volume stored due to the compression of the grains induced by an increase in the fluid pressure as well as the inter granular effective contact stress. The effect of inertia on the pore fluid momentum equation is considered to be insignificant within the frequency range for which the **u**–**p** approximation is valid [10], and hence this term is usually neglected. Combining Eq. (4) with Eq. (5) and applying the standard finite element procedure results in the second governing equation in following matrix form, i.e.,

$$\mathbf{L}^T\dot{\mathbf{u}} + \mathbf{S}\dot{\mathbf{p}} - \mathbf{H}\mathbf{p} = \mathbf{f}^p \tag{6}$$

where **S** and **H** are, respectively, the compressibility and flow matrices, and $\mathbf{f}^p$ is a fluid supply vector. Appendix A provides the details of these matrices and vectors.

Thus the global equations governing the behaviour of a saturated soil medium in a coupled **u**–**p** analysis can be written as

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}} \\ \ddot{\mathbf{p}} \end{bmatrix} + \begin{bmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{L}^T & \mathbf{S} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}} \\ \dot{\mathbf{p}} \end{bmatrix} + \begin{bmatrix} \mathbf{K} & \mathbf{L} \\ \mathbf{0} & -\mathbf{H} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} \mathbf{f}^u \\ \mathbf{f}^p \end{bmatrix} \tag{7}$$

3. Time integration

Finite element discretisation of the global equations leads to a system of second-order differential equations in which time is a continuous variable. In a direct time-integration scheme, Eq. (7) is integrated by a numerical step-by-step procedure. Therefore, equilibrium is only satisfied at discrete time intervals, and depending on which time steps are used to ensure the equilibrium, explicit and implicit integration methods can be distinguished. Although the computational cost of the implicit techniques can increase for complex problems compared to explicit methods, conditional stability of the explicit methods might be problematic for a coupled consolidation analysis. Hughes and Hilber [11] described the key characteristics that make a time marching scheme competitive and efficient. These include unconditional stability for linear problems, only one set of implicit equations to be solved at each time step, second order accuracy, controllable numerical dissipation at higher modes, self-starting, and no tendency of pathological overshooting of the true solution. Newmark's scheme is one of the most popular methods in the family of direct time integration techniques. In this method, the displacements and velocities at time t_{n+1} can be approximated by

$$\mathbf{u}_{n+1} = \mathbf{u}_n + \Delta t \dot{\mathbf{u}}_n + \frac{\Delta t^2}{2}[(1-2\beta)\ddot{\mathbf{u}}_n + 2\beta\ddot{\mathbf{u}}_{n+1}] \tag{8}$$

$$\dot{\mathbf{u}}_{n+1} = \dot{\mathbf{u}}_n + \Delta t[(1-\gamma)\ddot{\mathbf{u}}_n + \gamma\ddot{\mathbf{u}}_{n+1}] \tag{9}$$

where β and γ are Newmark's integration parameters.

The characteristics of Newmark's method depend on its integration parameters. This method is second-order accurate if $\gamma = 0.5$, otherwise it is only first-order accurate. Newmark's method attains numerical damping characteristics by selecting values larger than 0.5 for γ and choosing the smallest value for β compatible with