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Probabilistic analysis of numerical simulated railway track global stiffness

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ABSTRACT

The aim of this work is to assess numerically the influence of track geomaterials variability on the railway track stiffness. A non-intrusive probabilistic methodology based on *in situ* cone resistance tests is implemented in a 2D bidimensional finite element model with a modified plane strain condition. This model is used to estimate the track response to a moving load, which is characterized by the track stiffness measurement rolling stock. Spatial variability is taken into account by considering the cone resistance of each track layer as independent random fields, each one characterized by a marginal probability density function obtained from a statistical description of measured *in situ* data and a theoretical autocorrelation function. Despite input data variability, results presented much less variability than the input, which could be explained by both: load repartition over steppers, i.e. homogenization of the track stiffness measure, and deterministic characteristic of rail pads. Moreover, reduction of variance is observed, which means that less variance is observed for smaller correlation distances. In addition, a sensitivity analysis is also performed based on the Fourier Amplitude Sensitivity Test (FAST) and it showed, for the present case study, that the platform presents the highest first-order sensitivity index for all analyzed cases.

1. Introduction

Railway track geomaterials present different scales of complexity and heterogeneity. From the coarse grained ballast material to the platform, grain size, geometry and nature vary not only between layers but also inside each layer. Random variations of the mechanical properties of railway materials have been verified both *in situ* and in laboratory tests reported in the literature ([1–4]). Authors seem to agree that soil properties spatial variability may contribute to track degradation ([5,6]), although more *in situ* data in this sense seems necessary as to consolidate this result.

From a numerical point of view, the impact of material and load variability have been studied by authors under parametric (as when dealing with a track transition zone) ([5,7,8]) and perturbation approaches ([9]). Oscarsson [10] has proposed a probabilistic approach for characterizing the materials's variability, and more recently Rhayma et al. [11] used a probabilistic approach in order to verify the performance over maintenance criteria of different track maintenance operations. However, on both cases no spatial variability of soil properties is taken into account and in this sense railway track layers have been considered as perfectly homogeneous.

Random field theory provides a mathematical framework as to take into account spatial variability of a certain parameter following both a marginal probability density distribution and a correlation structure. The main features of this theory are developed on Section 2. It allows to take into account heterogeneities present on geomaterials in a probabilistic approach rather than parametric. In the literature, authors have successfully applied this methodology in order to perform reliability analysis on different geotechnical domains, such as slope failure ([12,13]), bearing capacity ([14]), excavation problems ([15]), seismic analysis ([16,17]), among others. However, little attention has been paid on the implications and capabilities of such analysis on the railway field.

In this paper, random field theory is applied on the railway field in order to establish a probabilistic characterization of the railway track global stiffness. Global track stiffness is considered as the ratio of the force applied through the wheel/rail contact and the rail displacement. This measure provides a good overview of the global response of the track to the train passage.

This paper is organized as follows: Section 2 describes the random field methodology and sensitivity analysis applied on this work; Section 3 describes the data obtained from *in situ* measurements performed in order to obtain a probabilistic description of the mechanical properties of railway materials; Section 4 presents the numerical model of the railway track used in this study; in Section 5 the probabilistic analysis of the track stiffness is presented

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and analyzed; a global sensitivity analysis is performed in Section 6 in order to evaluate which input contributed the most on the output's variance, and finally in Section 7 conclusions are drawn from the obtained results.

2. Theoretical background

The probabilistic non-intrusive methodology is applied in this study in order to characterize the railway track stiffness. This approach is largely used on risk analysis for different scientific and industrial domains ([18,19]). It is a robust and versatile method, as it allows to keep the model description in a classic deterministic way, and concentrate the uncertainties in input variables and their influence on the output variability. It can be presented as in Fig. 1 ([adapted from] [18]). Phase A consists on obtaining a numerical model which correctly reproduces the physical phenomena in question. This may be, in some cases, a simplified numerical model (or surrogate model) which was previously compared and validated with a more complex model or existing data. The model is often already available from previous developments. Phase B consists on obtaining a full probabilistic description of the input variables, in terms either of random variables or random fields if a spatial description is available or needed. This probabilistic description may be obtained either by available data (*in situ* or laboratory test), or from expert analysis and previous experience. This step is a crucial one for the probabilistic analysis. Results of numerical simulations will depend strongly on the input variability and correlations that may exist between them. Once this probabilistic description is obtained, Phase C consists on propagating the input's uncertainties through the model, in order to estimate a certain quantity of interest's variability. Moreover, failure analysis may be conducted based on existing or proposed failure thresholds. Finally, conducting both local and global sensitivity analysis may reveal which inputs impact the most the quantity of interest's variability. In this case, the sources of uncertainty may be reduced to only those that play an important role on the quantity of interest's variability.

2.1. Random fields

Many physical processes exhibit complex patterns of variation on both space and time. These may be characterized by random fields in order to completely describe the patterns of complex random phenomena. Random field theory establishes the basis on predicting, analyzing and decision making process under incomplete information about a given medium ([20]).

In this work only second-order invariant fields are considered. Non-Gaussian fields can be usually obtained from linear or non-linear translations of a Gaussian random field ([21]). All generating methods are based on the decomposition of the correlation structure. Three main classes of methods have been proposed in order to apply random fields on practical numerical applications: point discretization, shape function and series expansion methods. Point discretization methods ([22,17]) are among the most direct and easy methods to be used with a non-intrusive approach. Very often finite-element codes allow to give a certain value of the constitutive parameters at each cell, which is a direct transposition of the obtained values of the random field by a point discretization method. One important drawback is that this family of methods leads to discontinuities at element boundaries. Shape function methods have the advantage of ensuring a continuous description of the field over the elements, but these must be coded in the finite element code. Average discretization methods are based on weighted integrals of the random field, for which a better fit is expected due to the averaging process ([23]). Series expansion methods are probably the most complete representation of random fields, as both invariant and non-invariant fields can be equally represented. The Karhunen–Loève expansion is an example of this category. It is based on the covariance eigenfunction basis and is optimal in the sense of the mean square error resulting from the truncation after the Mth term ([18]). In this work, the midpoint method was chosen over the other methods because: (i) it is a non-intrusive approach and (ii) the random field is considered invariant and both the probability density function and the correlation structure are known and described by parametric functions.

It consists on the following steps:

1. Define a certain autocorrelation function. It can be either a theoretical function (cubic, exponential, squared exponential, triangular, etc.) or an identified structure from available data.
2. Obtain the autocorrelation matrix $[R]$ from the considered autocorrelation function of the random field. This matrix gives the correlation coefficient ρ_{ij} between the random variables X_i and X_j for any two locations y_i and y_j. It is represented on Eq. (1).

$$[R] = \begin{pmatrix} 1 & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & 1 & \cdots & \rho_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n1} & \rho_{n2} & \cdots & 1 \end{pmatrix} \tag{1}$$

3. Calculate the eigenvalues and eigenvectors of the autocorrelation matrix $[R]$. The eigenvectors are a independent uncorrelated basis on which the autocorrelation matrix is decomposed. The eigenvalues represent the variance of each component of this base.

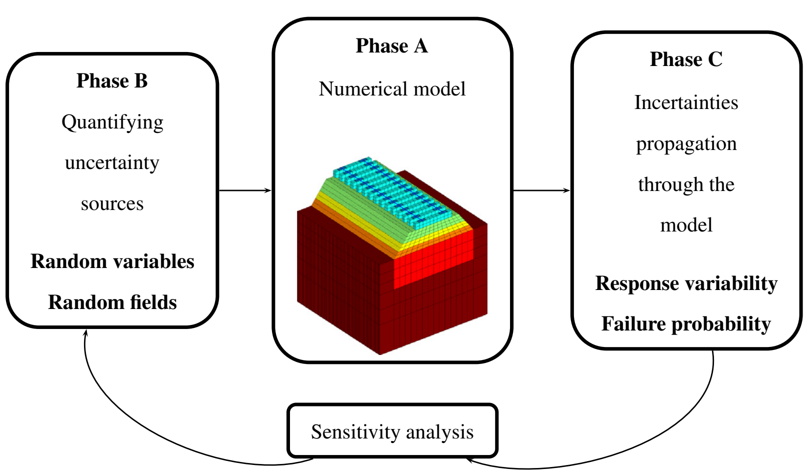

Fig. 1. Non-intrusive methodology, adapted from [18].