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Dual-phase coupled *u–U* analysis of wave propagation in saturated porous media using a commercial code

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ABSTRACT

This paper presents a method for incorporating dual-phase *u–U* wave propagation analysis for saturated porous media in commercial codes. Biot's formulation is first re-formulated in terms of effective stress and pore pressure, followed by its finite element discretization. The method of incorporating the dual-phase computation involves representing the solid and pore fluid phases by two overlapping meshes with collocated elements and nodal points. Viscous coupling between the solid and fluid phases are realized by connecting each pair of collocated nodal points using viscous Cartesian connectors. Volumetric compatibility between the solid and fluid phases is enforced by coding the behaviour of the two phases and their volumetric interaction within a user-defined material subroutine. Non-linear behaviour of the soil skeleton can also be prescribed within the same subroutine. Comparison with existing analytical or numerical solutions for three one-dimensional elastic problems shows remarkably good agreement. Finally, a three-dimensional elasto-plastic example involving impulsive surface loading on dry sand overlying saturated sand, which is akin to a dynamic compaction problem, is analyzed to demonstrate the potential application of the *u–U* analysis on a more realistic transient loading problem. The results of the analysis highlight the importance of the slow wave in dynamic compaction process. Although this approach was developed on the ABAQUS Explicit platform, it is, in principle, applicable to other existing codes with the right features. Its main advantage is that users can still access the computational functionalities and stability, as well as pre- and post-processing features that often come with commercial codes.

1. Introduction

Many soil dynamic problems such as blast loading and heavy tamping in saturated soils involve wave propagation through solid and fluid phases. Where the loading rate is much higher than the rate of drainage, an undrained condition is often assumed to prevail, in which there is no movement of pore fluid relative to the soil skeleton, so that the two phases are considered as a single phase [1]. However, this assumption may not be valid for all loading and ground conditions. Biot [2,3] showed that, even when there is no drainage of pore fluid, relative oscillatory motion can still exist between soil skeleton and pore fluid, and different dilatational waves propagate through pore fluid and soil skeleton at different speeds. In partially drained conditions, relative movement between soil skeleton and water will invariably occur. Hence, the soil skeleton and pore fluid is more accurately considered as two coupled phases in highly transient loading events. Truesdell's Mixture Theory [4,5] and Biot's dual-phase formulations [2,3] are commonly used to describe solid, fluid interaction under dynamic conditions. Of these two, Biot's formulation is the more widely used.

Biot's formulation considers the soil skeleton and pore fluid to be two phases coupled together by inertial, viscous and volumetric coupling. The primary variables of this u–U or u–U–π formulation are the displacement of the soil skeleton (u), displacement of the pore fluid (U) and partial fluid pressure (π). For problems involving lower loading rates, Zienkiewicz et al. [1,6] proposed that the inertial effect due to the relative acceleration between solid and pore fluid phases can be neglected. This leads to the u–p formulation, in which the primary variables are the displacement of the soil skeleton (u) and the pore pressure (p). The u–p formulation is sufficient for seismic problems; however, Zienkiewicz et al. [6] pointed out that it may be less accurate for high frequency, short duration problems. For such problems, a fully dynamic formulation, such as the u–U framework considered in this paper, is preferable.

Many dynamic soil–pore water interaction problems involving transient loading are still not well-studied. An example of this is the propagation of ground waves, such as that induced by explosion [7–9] or dynamic compaction [10–12], through full or partially saturated soils. Another example is excess pore pressure

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induced by breaking waves in rubble mound breakwaters [13–16] which may generate transient high wave loads on structures. Many of the numerical studies associated with explosion [7–9] used one-phase concept whereby the effect of fluid flow was not considered. Only [12] carried out a fully-coupled analysis of dynamic compaction using *u*–*p* formulation.

Existing analytical solutions for Biot's dual-phase formulation [17–21] are restricted to highly idealized one-dimensional problems of wave propagation through homogeneous linear elastic medium. Published numerical solutions are also largely limited to linear elastic behaviour and one dimensional idealization [22–27]. There are relatively few existing *u*–*U* codes and these, e.g. POROUS [17] and DYNAFLOW [28], tend to be codes developed in-house. Commercial codes, such as ABAQUS and LS-DYNA, are increasingly used in engineering analysis because they have well-developed 2- and 3-dimensional capabilities, portals for user-defined constitutive models, user interfaces and visualization tools. However, they do not have facilities for *u*–*U* analyses. This is possibly one reason for the dearth of *u*–*U* analyses.

Instead of developing a purpose-built *u*–*U* code, this paper proposes that *u*–*U* analysis can be implemented by incorporating appropriate user-defined subroutines into a commercial code, in this case, ABAQUS. By so doing, fully-coupled *u*–*U* dual phase analysis can be realized while retaining the functionality, user-interfaces and visualization tools of ABAQUS. This can potentially bring *u*–*U* dual phase analysis within the reach of many more engineers and researchers. This dual-phase coupled (DPC) approach is implemented on ABAQUS, but it can, in principle, be implemented on other commercial codes with similar user-defined portals.

In the discussion below, Biot's *u*–*U* formulation [2] is first summarized in terms of effective stress and pore pressure, following which its implementation using ABAQUS Explicit is presented. This will be followed by three validation examples involving one-dimensional wave propagation in saturated soil. Finally, one three-dimensional, non-linear, transient loading example is presented.

2. Formulations

2.1. Biot's u–U/u–U–π formulation

The original Biot's *u*–*U*/*u*–*U*–π formulation can be written in the form

$$\sigma^s_{ij,j} + c(\dot{U}_i - \dot{u}_i) - \rho_{11}\ddot{u}_i - \rho_{12}\ddot{U}_i = 0 \tag{1a}$$

$$\pi_{,j} - c(\dot{U}_i - \dot{u}_i) - \rho_{12}\ddot{u}_i - \rho_{22}\ddot{U}_i = 0 \tag{1b}$$

where σ^s_{ij} and π are the partial solid stress and partial fluid pressure, respectively, with positive stresses denoting tension; u and U are the solid and fluid displacement at time t; the single and double overdots denoting first and second derivatives with respect to time, respectively; c is the viscous damping coefficient; ρ_{11}, ρ_{12} and ρ_{22} are mass coefficients which relate the inertial forces to the acceleration in the solid and fluid phases.

The partial solid stress and fluid pressure are the average stresses on the solid and fluid phases respectively, and can be related to the effective stress σ'_{ij} and pore pressure p via the relations

$$\sigma_{ij} = \sigma'_{ij} - \delta_{ij}\alpha p = \sigma^s_{ij} + \delta_{ij}\pi \tag{2a}$$

$$p = -\pi/n \tag{2b}$$

in which σ_{ij} is the total stress, p the pore pressure (positive in compression), n the porosity, δ_{ij} the Kronecker delta and

$$\alpha = 1 - \frac{K'}{K_s} \tag{3}$$

where K' and K_s are the bulk moduli of the soil skeleton and soil grain, respectively. In most geotechnical problems, $K' \ll K_s$, so that $\alpha \sim 1$. Substituting Eq. (2) into Eq. (1), and assuming ρ_{12} to be negligible (to be discussed later), leads to Biot's *u*–*U* formulation in terms of effective stress and pore pressure, hereafter termed *u*–*U*–*p* formulation,

$$\sigma'_{ij,j} - (1-n)p_{,j} + c(\dot{U}_i - \dot{u}_i) - (1-n)\rho_s\ddot{u}_i = 0 \tag{4a}$$

$$-np_{,j} - c(\dot{U}_i - \dot{u}_i) - n\rho_f\ddot{U}_i = 0 \tag{4b}$$

in which ρ_s and ρ_f are the mass densities of the solid grain and pore fluid respectively. The viscous damping coefficient c is related to the Darcy's coefficient of permeability k and unit weight of the pore fluid γ_w by

$$c = \frac{n^2\gamma_w}{k} \tag{5}$$

The mass coefficients are related to the mass density of fluid and solid grains by the relations

$$\rho_{11} + \rho_{12} = (1-n)\rho_s \tag{6a}$$

$$\rho_{22} + \rho_{12} = n\rho_f \tag{6b}$$

The coefficient ρ_{12} is an inertial coupling coefficient which relates the stresses of one phase to the acceleration of the other. We can alternatively express the inertial interactions between the soil skeleton and fluid in Eq. (1) as

$$\sigma^s_{ij,j} + c(\dot{U}_i - \dot{u}_i) - (\rho_{11} + \rho_{12})\ddot{u}_i - \rho_{12}(\ddot{U}_i - \ddot{u}_i) = 0 \tag{7a}$$

$$\pi_{,j} - c(\dot{U}_i - \dot{u}_i) - (\rho_{22} + \rho_{12})\ddot{U}_i + \rho_{12}(\ddot{U}_i - \ddot{u}_i) = 0 \tag{7b}$$

If $u = U$, then two independent terms, namely ($\rho_{11} + \rho_{12}$) and ($\rho_{22} + \rho_{12}$), are sufficient to describe the inertial effects on the respective phases, and ρ_{12} on its own, which represents the advective inertial coupling arising from the relative fluid–solid motion, is not needed. The value of ρ_{12} is not readily determined and is often assumed to be zero (e.g. [17,18,23]).

The pore pressure is related to the strain tensor via

$$p = -K_f\frac{(\alpha - n)\varepsilon_{kk} + n\zeta_{kk}}{n + (1-n)\frac{K_f}{K_s} + \frac{K'K_f}{K_s^2}} \tag{8}$$

in which K_f is the bulk modulus of the fluid, ε_{kk} is the volumetric strain of the solid phase, and ζ_{kk} is the volumetric strain of the fluid phase, denoted as

$$\zeta_{kk} = U_{i,i} \tag{9}$$

In most saturated soils, $K' \ll K_f \ll K_s$, and Eq. (8) reduces to

$$p = -\frac{K_f}{n}[\varepsilon_{kk} - n(\varepsilon_{kk} - \zeta_{kk})] \tag{10}$$

If we further consider a completely "undrained" situation wherein the solid phase displacement and strain are the same as that of the pore fluid phase, then $\varepsilon_{kk} = \zeta_{kk}$ and Eq. (10) reduces to the standard form for pore pressure in an undrained situation, that is

$$p = -\frac{K_f}{n}\varepsilon_{kk} \tag{11}$$

2.2. Finite element discretization and time stepping

Following [6], application of the Galerkin method to Eq. (4) leads to the finite element form

$$\begin{bmatrix} \mathbf{K_s} + \mathbf{K_d} & \mathbf{K_{sf}} \\ \mathbf{K_{sf}} & \mathbf{K_f} \end{bmatrix}\begin{bmatrix} \bar{\mathbf{u}} \\ \bar{\mathbf{U}} \end{bmatrix} + \begin{bmatrix} \mathbf{H} & -\mathbf{H} \\ -\mathbf{H} & \mathbf{H} \end{bmatrix}\begin{bmatrix} \dot{\bar{\mathbf{u}}} \\ \dot{\bar{\mathbf{U}}} \end{bmatrix} + \begin{bmatrix} \mathbf{M_s} & 0 \\ 0 & \mathbf{M_f} \end{bmatrix}\begin{bmatrix} \ddot{\bar{\mathbf{u}}} \\ \ddot{\bar{\mathbf{U}}} \end{bmatrix} = \begin{bmatrix} \mathbf{f}^{(1)} \\ \mathbf{f}^{(2)} \end{bmatrix} \tag{12}$$