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Slip line theory applied to a retaining wall–unsaturated soil interaction problem

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ABSTRACT

An extension of slip line theory to unsaturated soils is presented and applied to the problem of a rigid retaining wall rotating about its toe into unsaturated soils. Suction is introduced using the effective stress concept. Soil–wall interface friction is defined carefully. The influence of suction on limiting passive earth pressures is analysed for two soils under steady state evaporation and infiltration. Suction increases the limiting passive stress at the soil–wall interface, with a dependence on the steady state flow type. The displacement of the retained soil is studied assuming the wall undergoes a rotation increment. The results show a clear difference in the displacement for evaporation and infiltration.

1. Introduction

In analyses of most earth pressure problems, for example retaining wall, slope stability and shallow footing problems, the soil is treated as being either completely saturated or completely dry. However, earth pressure problems often involve compacted soils or naturally formed soils above the ground water table. These soils are unsaturated and have an internal suction. Suction increases particle contact forces and makes unsaturated soils stronger than saturated or dry soils. This internal suction has not yet been incorporated into theoretical analyses of these problems in a rigorous way.

Earth pressure problems for saturated or dry soils are usually analysed by limit equilibrium methods [1–3] and slip line theory [4–6]. In the limit equilibrium approach, soil is treated as a rigid material and divided into blocks. Force and moment equilibrium equations are used to find the failure surface corresponding to the lowest factor of safety. On the other hand, slip line theory treats soil as a continuous body. A failure criterion is combined with the condition of stress equilibrium to form the governing equations for this limiting state. While the slip line theory is more restricted than the limit equilibrium methods in terms of geometries and boundary conditions, it is more amenable to realistic soil behaviours [7–9] and solutions of simple problems can be found in closed form.

In this paper, the slip line theory is extended to the case where soil is unsaturated. Suction is introduced into soil strength and stress state using the effective stress concept [10]. A steady state suction function [11] is combined with an expression for the effective stress parameter [12] to determine suction for a range of conditions. The soil is assumed to obey the Mohr–Coulomb failure criterion, coaxiality of stress and increment of strain, and dilate with a constant ratio of volumetric strain increment to shear strain increment.

The boundary value problem considered is a rigid retaining wall interacting with unsaturated soil. The governing stress and displacement equations are discretised by the finite difference method and the numerical procedures used were first calibrated with comparable data from the literature. Typical mechanical parameters and flow parameters for two soils (an unsaturated sand and an unsaturated silt) in steady state evaporative and infiltrative conditions are chosen to compute the effective stresses at the soil–wall interface. The initial displacement of retained soil is also studied by assuming the wall at the limiting passive condition undergoes a rotation increment, with rotation occurring about the toe. The displacement slip line field is obtained from its stress equivalent. An interface slip angle is incorporated into the analysis to allow for soil slippage at the soil–wall interface.

It is shown that the existence of suction can greatly increase the limiting passive stresses at the soil–wall interface. The computed limiting passive stresses reduce to results given by Rankine theory adapted to include a suction dependence when the interface friction is zero. Also, steady state variation in suction, for example

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Nomenclature

List of symbols

- s soil suction
- u_a, u_w pore air, pore water pressures
- s_e soil suction value separating saturated from unsaturated states
- s_{ei} interface suction value separating saturated from unsaturated states
- λ inverse of s_e
- χ effective stress parameter
- ϕ' soil friction angle
- δ'_s surface friction angle
- δ'_i interface friction angle
- ψ soil dilation angle
- ψ_i interface slip angle
- c' soil cohesion
- c'_i interface cohesion
- γ_t, γ_w soil unit weight and water unit weight
- k_s saturated hydraulic conductivity
- q steady state flow rate
- σ_1, σ_3 major and minor principal stresses
- σ_{xx}, σ_{yy}, σ_{nn} normal stresses
- σ_{xy}, σ_{yx}, σ_{nt} shear stresses
- σ'_0 scaling stress
- L length of retaining wall
- l distance from toe to a point on retaining wall
- H depth of water table
- p surcharge
- K_p passive earth pressure coefficient
- θ angle between vertical axis and major principal stress direction
- η, ξ, α, β families of curves
- μ_s angle between η, ξ curves and the major principal stress direction
- μ_v angle between α, β curves and the major principal strain direction
- ω wall rotation angle
- $\delta\omega$ rotation angle increment
- ε_1, ε_3 major and minor normal strains
- ε_{xx}, ε_{yy}, ε_{xy}, ε_{yx} strain components
- γ engineering shear strain
- v volumetric strain
- U_x, U_y, U_α, U_β, U_n, U_t displacement in the x, y, α, β, n, t directions. The prime symbol (′) attached to a stress indicates it is an effective stress

due to infiltration or evaporation, may significantly change the limiting passive stresses on the wall. Furthermore, the results show clear difference in the initial soil displacement for steady state evaporative and infiltrative conditions.

2. Effective stress

Informed by experimental evidence, Bishop [10] extended Terzaghi's effective stress to unsaturated soils:

$$\sigma' = \sigma - u_a + \chi(u_a - u_w) \tag{1.1}$$

where $\chi \equiv$ effective stress parameter, $u_a \equiv$ pore air pressure and u_a - $\equiv$ pore water pressure. The pore air pressure is hereafter assumed to be zero (assumed datum for atmospheric pressure) so the effective stress in Eq. (1.1) becomes:

$$\sigma' = \sigma + \chi s \tag{1.2}$$

where $s = (u_a - u_w)$ is the soil suction. Similar expressions have been adopted by others [13–15]. The effective stress parameter is influenced by many factors such as soil type, whether the soil is undergoing a drying or wetting cycle, the loading history leading to a particular value of degree of saturation [16] and particular soil structure [17]. Many expressions have been proposed for estimating the effective stress parameter. One of the most appealing is that proposed by Khalili and Khabbaz [12]:

$$\begin{cases} \chi = (s/s_e)^{-0.55}, & s \geqslant s_e \\ \chi = 1, & s < s_e \end{cases} \tag{2}$$

where $s_e \equiv$ suction value separating saturated from unsaturated states.

Although χ in Eq. (2) contains no volumetric parameter, Eq. (2) and its variations [18,19] were shown to be able to predict effective stress for a wide range of soils on both mechanical and hydraulic stress paths. One advantage of Eq. (2) is its simplicity. No more than the one parameter, i.e. the suction value separating saturated from unsaturated states, is required and it can be obtained in a soils laboratory. Eq. (2) is therefore adopted here for estimating the contribution of suction to the effective stress.

3. Soil model and interface model

The analysis in this paper assumed a Mohr–Coulomb failure criterion (Fig. 1) for both unsaturated soil and the unsaturated soil–wall interface, expressed respectively as:

$$(\sigma'_1 - \sigma'_3)/2 = [(\sigma_1 + \sigma_3)/2 + \chi s + c' \cot\phi'] \sin\phi' \tag{3}$$

where $\sigma'_1, \sigma'_3 \equiv$ major and minor effective principal stresses, respectively, $c' \equiv$ soil cohesion and $\phi' \equiv$ soil friction angle, and:

$$(\sigma'_1 - \sigma'_3)/2 = [(\sigma_1 + \sigma_3)/2 + \chi s + c'_i \cot\delta'_i] \sin\delta'_i \tag{4}$$

where $c'_i \equiv$ interface cohesion and $\delta'_i \equiv$ interface friction angle.

It is assumed that the same friction angle is mobilised at every point in the unsaturated soil. The magnitude of this friction angle can be significantly higher than the critical state friction angle, especially under low confining pressures. Other models where friction angle is dependent on stress have been used [20–22];

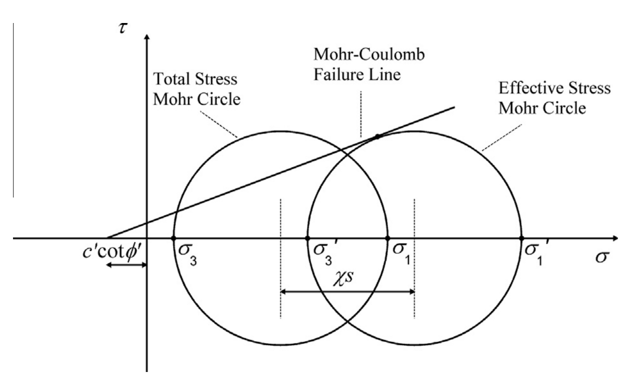

Fig. 1. Graphical representation of the Mohr–Coulomb failure envelope (total stress Mohr circle is obtained by shifting effective stress Mohr circle a distance χs to the left).