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Distribution of the factor of safety, in geotechnical engineering, for independent piecewise linear capacity and demand density functions

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ABSTRACT

In many geotechnical engineering cases, the factor of safety may be defined as the ratio of the capacity, of the geotechnical structure or its support elements, to the pertinent demand. By representing the capacity and the demand as independent piecewise linear random variables, an analytic solution is obtained for the probability density and cumulative distribution functions of the factor of safety. Thus, solutions for the calculation of the mean value, the standard deviation and the minimum and maximum values of the factor of safety, are provided. Application of the developed analytical solutions, to the probabilistic analysis of a published case of rock spalling in a deposition tunnel complex, follows. The methodology allows for the parametric evaluation of the effect of specific design variables to the distribution of the safety factor and to the probability of failure. The closed form solution may be programmed as a computer code that may run easily on a tablet or netbook or even on a smartphone. It proves useful for the probabilistic design of a variety of geotechnical applications, such as foundations, tunneling, mining, underground roof reinforcement, and earth retaining structures, and permits decisions to be taken in terms of risk and reliability.

1. Introduction

Failure of a system is assessed by its inability to perform its intended function adequately on demand. The opposite, the measure of success, is called reliability. Failure is qualitative, whereas reliability can be defined and quantified, as the probability of an object to perform its required function adequately under stated conditions for a specified period of time. The purpose of reliability based design is to produce an engineering system whose failure would be an event of very low probability. The acceptance of a level of reliability must be viewed within the context of possible costs, risks and associated social benefits.

Failure is regarded as a random phenomenon. Reliability prediction combines the formulation of a proper reliability model together with the estimation of the model input parameters to provide a system level estimate for the output reliability parameters. Probabilistic design deals primarily with the consideration of the effects of random variability upon the performance of an engineering system. It is a tool that is mostly used in areas that are concerned with quality and reliability. It differs from classical approaches to design, as it assumes a small probability of failure instead of using a conventional safety factor. When using a probabilistic approach to design, each variable has not a single value, but is viewed as a probability distribution. Essentially, probabilistic design focuses upon the prediction of the effects of random variability. Methods that are used to predict the random variability of an output include: Monte Carlo, propagation of error, design of experiments (DOE), statistical interference. Reliability based analysis for geotechnical engineering design, has been tackled, amongst others, by Harr [1], Pine [2], Skipp [3], Hoek et al. [4], James [5], and Hoek [6]. A complete reference to probabilistic methods for geotechnical analysis is given by Baecher and Christian [7]. Modern numerical tools like finite elements and neural networks combined with probabilistic analysis rationalize the design as presented by Deng et al. [8]. With the application of elastoplastic finite elements, and the random field theory, Griffiths et al. [9] and Griffiths and Fenton [10] account for both the variability of the rock and soil properties and the spatial correlation. The growing availability of computational tools and power, allows for the employment of such methods for the endeavor of reliability analyses.

1.1. Factor of safety

In geotechnical engineering, assessments of the risk of failure are made on the basis of allowable factors of safety, learned from previous experiences (e.g. [11–14]) for a given system in its anticipated environment. The factor of safety, is a term describing the structural capacity of a system beyond the expected loads or

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actual loads; essentially, how much stronger the system is than it usually needs to be for an intended load. Engineering systems are purposefully built much stronger than needed for normal usage to allow for emergency situations, unexpected loads, misuse, or degradation. The strength needed is rationally evaluated through modeling, i.e. the idealization of a structure, which admits to simple but logical mathematical solution and still contains the essential elements of the prototype. There, induced loadings are commonly modeled by completely defined, simple analytical or geometric representations. Material characterization is taken to be complete, and inherent properties are assumed to be stable and uniquely defined. However, geometric configurations and inherent material properties are incompletely known, with uncertainties such as in: soil and rock strength, structure, alteration, seepage, natural stress, dynamic actions, freezing and thawing, environmental factors, workmanship, microcrystalline imperfections, induced loadings, and material properties.

The factor of safety fs is defined as the ratio of the capacity C of the object upon the demand D, and failure is taken to occur when it is less than one. This is equivalent to a safety margin larger than zero. In general, the demand function is the resultant of many uncertain components of the system under consideration, mainly spatial or geological, and similarly the capacity function will depend on the variability of engineering material parameters, testing errors, construction procedures, inspection supervision, ambient conditions, etc.

1.2. Analytical formulation

The capacity C and the demand D may be considered as random variables, with probability density functions (PDF), f_C and f_D, respectively. A safety formulation may be formed in the form of either the safety margin M, defined as the difference between capacity and demand, or the factor of safety fs, defined as the ratio of capacity to demand, i.e.:

$$fs = \frac{C}{D}; \quad M = C - D = D \cdot (fs - 1) \tag{1}$$

By definition fs is also a random variable with cumulative distribution function (CDF) F_{fs}. For $C > 0$ and $D > 0$, F_{fs} is defined as:

$$F_{fs}(fs) = P\left(\frac{C}{D} < fs\right) = P\left(D > \frac{C}{fs}\right) \tag{2}$$

It is calculated by the integral of the joint (bivariate) PDF $f(C,D)$, of the variables C and D, over the domain S_{fs} on the C–D plane, such that $C/D \leqslant fs$.

$$F_{fs}(fs) = \iint_{S_{fs}} f(C,D)dDdC \tag{3}$$

In most of such analyses, the random variables C and D are assumed to have infinite extent and usually to be normally distributed. Negative or infinite extents are unreasonable for most geotechnical problems, and therefore truncations are necessary to retain the values of the variables within realistic limits.

If the random variables C and D are independent, then the joint PDF is simply the product of the PDF of the two random variables. Then (3) can be written as:

$$F_{fs}(fs) = \iint_{S_{fs}} f_C(C) f_D(D) dDdC \tag{4}$$

If the domains of f_C and f_D are $[L_C, U_C]$ and $[L_D, U_D]$, respectively, F_{fs} is calculated by:

$$F_{fs}(fs) = \int_{L_C}^{\min\{U_C, U_D fs\}} \int_{\max\{L_D, \frac{C}{fs}\}}^{U_D} f_C(C) f_D(D) dDdC \tag{5}$$

Breaking apart the integration limits, Eq. (5) can be rewritten as the sum of two double integrals:

$$\begin{aligned} F_{fs}(fs) = & \int_{L_C}^{\max\{L_C, L_D fs\}} f_C(C)\left[\int_{L_D}^{U_D} f_D(D)dD\right]dC \\ & + \int_{\max\{L_C, L_D fs\}}^{\min\{U_C, U_D fs\}} f_C(C)\left[\int_{C/fs}^{U_D} f_D(D)dD\right]dC \end{aligned} \tag{6}$$

In Eq. (6) the integral $\int_{L_D}^{U_D} f_D(D)dD$ is the cumulative probability over the whole domain of D so it equals 1 and the integral $\int_{C/fs}^{U_D} f_D(D)dD$ can be written as $1 - F_D(C/fs)$. Hence, (6) may be simplified as:

$$F_{fs}(fs) = F_C(R_U) - \int_{R_L}^{R_U} f_C(C) \cdot F_D(C/fs)dC \tag{7}$$

where F_C and F_D are the CDFs of C and D respectively, and

$$\begin{aligned} R_L = R_L(fs) = L_D \cdot \max\left\{\frac{L_C}{L_D}, fs\right\} \text{ and } R_U = R_U(fs) \\ = U_D \cdot \min\left\{\frac{U_C}{U_D}, fs\right\} \end{aligned} \tag{8}$$

Thus, four segments (α, β, γ, δ) may be distinguished for fs, according to its position relative to the ratios L_C/L_D and U_C/U_D. These segments for fs and their pertinent pairs of the R_L, R_U functions are given in Table 1.

2. Piecewise linear density functions

Straight line density functions are not common in defining random variables, as they generally lack the ability to model appropriately nonlinearly distributed random variables. This, may be overcome by the employment of piecewise linear density functions (e.g. triangular, polygonal), already suggested, by Biernatowski and Puła [15] and Puła and Traczyk [16], as useful for safety evaluations in geotechnics. Although such distributions devoid smoothness, this handicap may be mitigated, if desired, by increasing the number of their segments. Moreover, they allow for analytical treatment.

Table 1
Relative positions of fs.

fs	$\leqslant U_C/U_D$	$\geqslant U_C/U_D$	R_L=
$\leqslant L_C/L_D$	α	δ	L_C
$\geqslant L_C/L_D$	γ	β	$L_D \cdot fs$
R_U=	$U_D \cdot fs$	U_C	

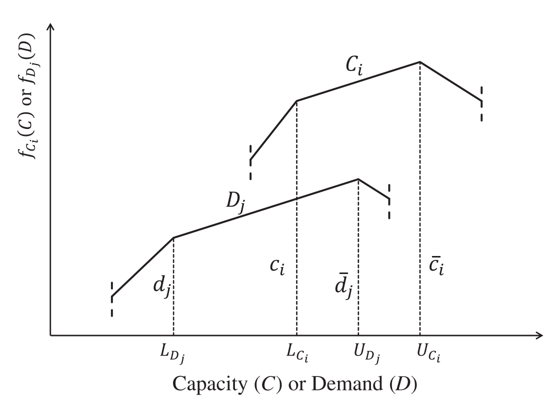

Fig. 1. Probability density function segments for piecewise linear capacity C and demand D.