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Finite volume coupling strategies for the solution of a Biot consolidation model

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ABSTRACT

In this paper a finite volume (FV) numerical method is implemented to solve a Biot consolidation model with discontinuous coefficients. Our studies show that the FV scheme leads to a locally mass conservative approach which removes pressure oscillations especially along the interface between materials with different properties and yields higher accuracy for the flow and mechanics parameters. Then this numerical discretization is utilized to investigate different sequential strategies with various degrees of coupling including: iteratively, explicitly and loosely coupled methods. A comprehensive study is performed on the stability, accuracy and rate of convergence of all of these sequential methods. In the iterative and explicit solutions four splits of drained, undrained, fixed-stress and fixed-strain are studied. In loosely coupled methods three techniques of the local error method, the pore pressure method, and constant step size are considered and results are compared with other types of coupling methods. It is shown that the fixed-stress method is the best operator split in comparison with other sequential methods because of its unconditional stability, accuracy and the rate of convergence. Among loosely coupled schemes, the pore pressure and local error methods which are, respectively, based on variation of pressure and displacement, show consistency with the physics of the problem. In these methods with low number of total mechanical iterations, errors within acceptance range can be achieved. As in the pore pressure method mechanics time step increases more uniformly, this method would be less costly in comparison with the local error method. These results are likely to be useful in decision making regarding choice of solution schemes. Moreover, the stability of the FV method in multilayered media is verified using a numerical example.

1. Introduction

The coupled process of fluid flow and mechanics in geotechnics was first introduced by Terzaghi in 1924 as a consolidation phenomenon. This theory described one dimensional consolidation analytically and has since been widely used in practice to calculate ground settlements [1]. Subsequently in 1941 Biot generalized Terzaghi's theory to three-dimensional porous media based on a linear stress–strain constitutive relationship and a linear form of Darcy's law [2]. The fluid flow-stress analysis in porous media is of increasing importance today in a diverse range of engineering fields included reservoir engineering, biomechanics, and environmental engineering [3].

As the solutions of Biot system in closed forms are only available in special cases, numerical methods are commonly used for solving the respective initial-boundary value problem. However, numerical approximations based on different forms of the governing equations and numerical methods can lead to significantly different results for the cases of non-linear flow equations in porous media [4,5]. In spite of extensive research that has been carried out for the numerical solution of the Biot equations, there still exist challenging issues which are as follow. First is the instabilities that occur because of sharp transient gradients. One of the numerical methods that suffers from this kind of instability is the standard finite element method (FEM). The FEM is widely used in solving poroelasticity systems, especially in the cases when dealing with complex geometry or adaptive grids [1,6–8]. Although the standard FEM provides accurate results for the problems with smooth solutions, when strong pressure gradients appear, these methods may not be stable in the sense that strong nonphysical oscillations occur in the approximation of the pressure field [9]. To avoid these difficulties, a staggered finite difference discretization for the poroelasticity equations in a single layer was examined by Gaspar et al. [10]. The approach from [10] was further developed by Naumovich

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Nomenclature

$\boldsymbol{b}$	body force (M L^{-2} T^{-2})
c	consolidation coefficient (L^2 T^{-1})
c_{br}	solid grain compressibility (M^{-1} L T^2)
c_M	vertical uniaxial compressibility (M^{-1} L T^2)
er	relative error of local error method
er_g	goal local error
f	flow source or sink (T^{-1})
g	gravitational acceleration (L T^{-2})
h	size of control volume (L)
$\bar{\mathbf{K}}$	hydraulic conductivity tensor (L T^{-1})
K_u	undrained bulk modulus (M L^{-1} T^{-2})
L	length of domain (L)
m	fluid content
M	Biot modulus (M L^{-1} T^{-2})
N	number of nodes
p	fluid pore pressure (M L^{-1} T^{-2})
P_L	overburden (M L^{-1} T^{-2})
t	time (T)
T	time interval (T)
$\mathbf{u}$	displacement (L)
$\mathbf{v}$	Darcy's velocity (L T^{-1})
α	Biot coefficient
β	fluid compressibility (M^{-1} L T^2)
Γ	frontier of the domain of coupled partial differential equations
$\boldsymbol{\varepsilon}$	strain
$\in$	acceptable tolerance for the L^2-norm of two iterative solutions in one time step
θ	time weighted factor
κ	harmonic averaging of $\bar{\mathbf{K}}/\rho g$ over the interval (x_{i-1}, x_i) (M^{-1} L^3 T)
λ	Lame constant (M L^{-1} T^{-2})
μ	shear modulus (M L^{-1} T^{-2})
v	harmonic averaging of $\lambda + 2\mu$ over the interval $(x_{i-0.5}, x_{i+0.5})$ (M L^{-1} T^{-2})
ξ	interface (L)
ρ	fluid density (M L^{-3})
$\boldsymbol{\sigma}$	effective stress (M L^{-1} T^{-2})
σ_t	total mean stress (M L^{-1} T^{-2})
τ	time step size (T)
ϕ	porosity
χ	spatial weighted factor for location of interface
$\bar{\omega}_p$	spatial grids for pressure
$\bar{\omega}_u$	spatial grids for displacement
ω_T	time grids

et al. [11] to the case of multilayered deformable porous media, based on finite volume discretization. The order of convergence of this scheme is analyzed by Ewing et al. [12].

One of the numerical methods that is widely used in computational fluid dynamics problems is finite volume method (FVM). The main advantage of the FVM is that, it produces accurate discretization for systems of partial differential equations with discontinuous coefficients. The corresponding schemes yield local mass conservation and lead to higher accuracy for stresses and fluxes at the interfaces [13]. Therefore, it is worthwhile to examine FVM for addressing the instability problem in numerical solution of poroelasticity systems.

A second challenge is related to the coupling strategies. There are four types of strategies for solving the coupled flow and mechanics problem: fully coupled, iteratively coupled, explicitly coupled and loosely coupled. In the fully coupled method, the governing equations of flow and mechanics are solved simultaneously at each time step [14–16] but in the iterative approach, by partitioning the coupled problem, one of the flow or mechanical sub-problem is solved first and then the other is solved using the intermediate solution information. This procedure iterates at each time step until the solutions converge to the fully coupled approach [17–20]. Explicitly coupled is a kind of iterative scheme where only one iteration is considered [21]. In loosely coupled, the mechanics equation is not solved in each time step [22] and after multiple flow steps are taken, the solution of the mechanical sub-problem is updated. In this method the number of flow steps depends on the change of the pore-pressure or displacement and the specified allowable error [22].

The fully coupled approach is unconditionally stable but it leads to a large algebraic system that requires huge computational efforts for large problems. This linear system may be severely ill-conditioned [23]. To resolve these problems, three sequential methods including, iteratively, explicitly and loosely coupled methods are employed. In addition, sequential methods have the advantage of using existing flow and geomechanics simulators. Various sequential methods are different in the stability, accuracy and efficiency behaviors [24]. The loosely coupled approach is an inexpensive method compared with other types of coupling but the iterative and then the explicit methods exhibit higher accuracy than this approach [25]. Due to the need for balance between accuracy and efficiency, comprehensive investigations of these strategies are performed in this study.

In the iterative methods, based on the partitioning strategy, different operator splits are considered. In the splits where the mechanical equation is solved first, the drained and undrained splits are applied. In a drained split, it is assumed that there is no pressure change in mechanical equation and this yields conditional stability. On the other hand, in an undrained split, fluid mass remains constant during the mechanical step. The undrained split is unconditionally stable except for an incompressible system in which this operator split is not convergent [17]. In other iterative schemes, which have been applied in reservoir engineering, the flow problem is solved first [25,26]. Fixed-strain and fixed-stress methods are two operator splits of this type of sequential schemes. In a fixed-strain split the rate of total volumetric strain is considered constant during the flow calculation while in a fixed-stress split the rate of total volumetric stress is a constant parameter. In a fixed-strain split conditional stability occurs but a fixed-stress split provides unconditional stability even if the system is incompressible [18]. Examples of models based on the iterative coupled approaches are given by Jha and Juanes [19] who have investigated sequential schemes by employing a FEM for the mechanical problem and a mixed FEM for the flow problem. Also Kim et al. analyzed stability and convergence of iterative methods based on a FEM and a FVM for the mechanical and the flow problems, respectively [24,17,18].

In the explicit approach, there is no iteration in each time step to ensure convergence of the solution and there is only single pass between mechanics and flow equations. As iterative methods, one can use all operator splits described above: the drained, undrained, fixed-strain and fixed-stress splits [24].