Computers and Geotechnics 38 (2011) 363–374

Contents lists available at ScienceDirect

Computers and Geotechnics

journal homepage: www.elsevier.com/locate/compgeo

Probabilistic analyses of a strip footing on horizontally stratified sandy deposit using advanced constitutive model

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ARTICLE INFO

Article history:
Received 12 August 2010
Received in revised form 29 November 2010
Accepted 30 December 2010
Available online 3 February 2011

Keywords:
Probabilistic methods
Constitutive models
Random fields
Foundation settlement

ABSTRACT

An advanced hypoplastic constitutive model is used in probabilistic analyses of a typical geotechnical problem, strip footing. Spatial variability of soil parameters, rather than state variables, is considered in the study. The model, including horizontal and vertical correlation lengths, was calibrated using a comprehensive set of experimental data on sand from horizontally stratified deposit. Some parameters followed normal, whereas other followed lognormal distributions. Monte-Carlo simulations revealed that the foundation displacement u_y for a given load followed closely the lognormal distribution, even though some model parameters were distributed normally. Correlation length in the vertical direction θ_v was varied in the simulation. The case of infinite correlation length was used for evaluation of different approximate probabilistic methods (first order second moment method and several point estimate methods). In the random field Monte-Carlo analyses with finite θ_v, the vertical correlation length was found to have minor effect on the mean value of u_y, but significant effect on its standard deviation. As expected, it decreased with decreasing θ_v due to spatial averaging of soil properties.

1. Introduction

Parameters of simple constitutive models, which are typically used in probabilistic analyses of geotechnical problems, are dependent on the soil state. These models thus do not allow us to distinguish whether the measured variability of soil properties is caused by the variability of soil type or soil state. Contrary to this, advanced constitutive models adopt soil parameters that are specific to the given soil granulometry and mineralogical composition of soil particles. State variables (such as void ratio e) then incorporate the state-dependency of the soil behaviour. In this respect, the sources of the objective (aleatory [25]) uncertainty in soil mechanical behaviour can be subdivided into two groups:

1. In some situations, soil mineralogy and granulometry may be regarded as spatially invariable, and the uncertainty in the mechanical properties of soil deposit come from variability in the soil state. In this case, soil *parameters* of advanced models may be considered as constants, and in the analyses it is sufficient to consider spatial variability of *state variables* describing the relative density of soil.
2. In other cases, soil properties are variable due to varying granulometry and mineralogy of soil grains. Such a situation is for example typical for soil deposits of sedimentary basins, where the granulometry varies due to the variable geological conditions during the deposition. In such a case, it is necessary to consider spatially variable soil *parameters* in the simulations.

Application of advanced constitutive models within probabilistic numerical analyses still remains relatively uncommon in the geotechnical scientific literature. Moreover, most of the applications of probabilistic methods in combination with advanced soil constitutive models consider the uncertainty in the state variable only, while keeping constant values of the model parameters. As an example, Hicks and Onisiphorou [27] studied stability of underwater sandfill berms. Their aim was to study whether presence of 'pockets' of liquifiable material may be enough to cause instability in a predominantly dilative fill. They used a double-hardening constitutive model Monot [33] with probabilistic distribution of the Been and Jefferies [3] state variable ψ. As the aim of the research was to study whether pockets of loose material may cause failure of the berm, the approach chosen (variation in the state variable only) is fully justifiable. In other applications, Tejchman [45] studied the influence of the fluctuation of void ratio on formation of the shear zone in the biaxial specimen using the hypoplastic model by von Wolffersdorff [47]. Similar procedure and the same constitutive model was used by other researchers in finite element simulations of different geotechnical problems [34,36]. Finally, Andrade et al. [1] considered random porosity fields in combination with an advanced constitutive model and studied their influence on strength and shear band formation in a biaxial specimen.

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doi:10.1016/j.compgeo.2010.12.007

The goal of this paper is to present a complete evaluation of the influence of parameter variability of an advanced constitutive model and its influence on predictions of a typical geotechnical problem. To utilise the advantage of the non-linear formulation of the constitutive model adopted, we study settlement of a rigid strip foundation subjected to a given load. While most application of probabilistic methods to the foundation problems focus on the evaluation of the bearing capacity [17,15,7,11,28,29,21,13], less attention is payed to the quantification of the uncertainty in serviceability limit states. In the available studies, the authors focus on different aspects of the problem, such as foundation size and geometry [30], uncertainty in the foundation load [5], 3D effects [19,14], differential settlement issues between two footings [12,14,34], cross-correlation between elastic parameters E and ν [35], the effects of layers of different materials in the subsoil [32], and comparison with simpler probabilistic methods (such as the first order second moment method) [20]. In many publications, the authors address the influence of the correlation length [35,14,19,12]. In most of these works, however, the soil is modelled as a linear elastic material (or elastic material with stress-dependent Young modulus [30]). An exception is the contribution by Niemunis et al. [34], who used non-linear hypoplastic model with constant parameters and random fields of void ratio in the cyclic analyses of two adjacent strip footings. Most of the available studies thus do not consider the non-linear soil behaviour, which is important for correct predictions of foundation displacements. This issue is addressed in the present work.

2. Experimental program

The material for the investigation comes from the south part of upper Cretaceous Třeboň basin in south Bohemia (Czech Republic) from the sand pit Kolný [42]. The pit is located in the upper part of the so-called Klikovské layers, youngest (Senon) strata of the south Bohemian basins. These fluvial layers are characterised by a rhythmical variation of gravely sands, sands and clayey sands.

Altogether forty samples were obtained from a ten meters high pit wall in a regular rectangular grid (Fig. 1). The laboratory program was designed to provide for each of the samples enough information to calibrate the hypoplastic model for granular materials by von Wolffersdorff [47]. The following tests were performed on each of the 40 samples:

- Oedometric compression tests on initially very loose specimens with loading steps 100, 200, 400, 800, 1600, 3200 and 6400 kPa.
- Drained triaxial compression test on specimen dynamically compacted to void ratio corresponding to the dense in situ conditions. One test per specimen at the cell pressure of 200 kPa.
- Measurement of the angle of repose.

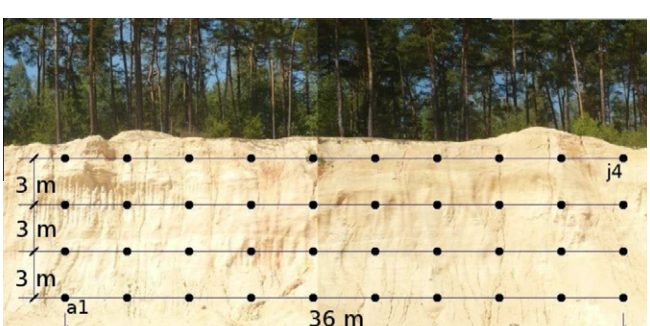

Fig. 1. The wall of the sand pit in south part of the Třeboň basin. Black dots represent positions of specimens for the laboratory investigation.

Results of all the laboratory experiments are presented in the Appendix. Location of the specimens, labeled as $a1$–$j4$, is indicated in Fig. 1. Note that four specimens ($c1$, $e4$, $f1$, $f2$) showed unusual behaviour, and these specimens were not used in the evaluation.

In addition to laboratory experiments, five in situ porosity tests with membrane porosimeter were performed at different locations within the area from which the samples were obtained. Average natural void ratio was 0.41. The porosity was found to be fairly uniform and the sand was in very dense conditions. Note that the triaxial tests were not performed at the initial void ratio exactly corresponding to the in situ conditions, as this was not known for each of the 40 samples. This fact should, however, not influence the model calibration, as parameters of advanced hypoplastic models depend on soil type and granulometry only, and do not significantly depend on its state [26,24].

3. Calibration of hypoplastic constitutive model

The constitutive model selected for this research work is based on hypoplasticity, a particular class of incrementally non-linear constitutive models. The hypoplastic equation may be written as

$$\overset{\circ}{\mathbf{T}} = f_s \mathcal{L} : \mathbf{D} + f_s f_d \mathbf{N} \|\mathbf{D}\| \tag{1}$$

where $\overset{\circ}{\mathbf{T}}$ is the objective (Jaumann) stress rate, $\mathbf{D}$ is the Euler's stretching tensor and $\mathcal{L}$ and $\mathbf{N}$ are fourth- and second order constitutive tensors, respectively. f_s and f_d are scalar factors expressing the influence of the stress level (barotropy) and density (pyknotropy). The model adopted in this research was proposed by von Wolffersdorff [47] based on the earlier work of the Karlsruhe research group (e.g., [31,22,2]). For an interpretation of the model response see [23].

The hypoplastic model by von Wolffersdorff [47] has eight material parameters, namely φ_c, h_s, n, e_{d0}, e_{c0}, e_{i0}, α and β. Their calibration procedure was detailed by Herle and Gudehus [26]. A somewhat simplified calibration procedure has been adopted in the present work. The whole process of calibration has been automated to reduce subjectivity of calibration.

The critical state friction angle φ_c has been obtained directly by the measurement of the angle of repose. The hypoplastic model considers that the soil state in the e vs. p space is bound by maximum (e_i) and minimum (e_d) void ratios, as shown in Fig. 2. In addition, critical state line in the e vs. p space is characterised by void ratio e_c. The three curves are described by formula due to Bauer [2]

$$\frac{e_c}{e_{c0}} = \frac{e_d}{e_{d0}} = \frac{e_i}{e_{i0}} = \exp\left[-\left(\frac{3p}{h_s}\right)^n\right] \tag{2}$$

with five parameters. The parameter n controls the curvature of the curves and h_s controls the overall slope of the curves. The parame-

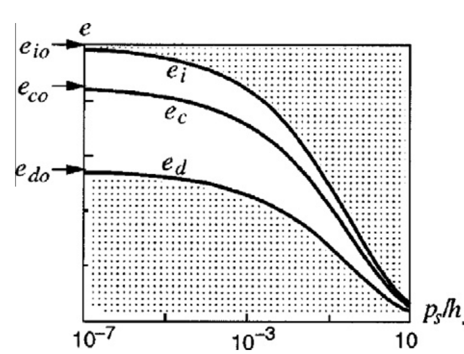

Fig. 2. The dependency of the reference void ratios e_d, e_c and e_i on the mean stress [26].