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Assessment of the concrete strength in existing buildings using a finite population approach

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HIGHLIGHTS

- A new method to assess concrete strength in existing building is presented.
- The method disaggregates the concrete variability into finite populations.
- The CoV of the concrete strength is evaluated using the CoV of rebound hammer tests.
- The method controls the uncertainty in the estimate of concrete strength variability.
- The method controls the uncertainty in the estimate of the mean concrete strength.

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ABSTRACT

A framework is defined to evaluate the concrete compressive strength in existing buildings and control the uncertainty associated to the survey planning and to the concrete strength randomness. The framework proposes the discretization and disaggregation of the concrete strength in a building into finite populations of elements. Finite population statistics are used to correlate the number of tests performed in each population with the uncertainty about the mean and the coefficient of variation (CoV) of the concrete strength. A method to estimate the CoV of the concrete strength using the CoV of rebound hammer test results is also proposed to overcome the need for a high number of destructive tests. Results show that the proposed approach effectively controls the uncertainty in the estimate of the variability of the concrete strength in a population as well as the uncertainty in the estimate of the mean value of the concrete strength.

1. Introduction

In the safety assessment of existing buildings, quantifying the “as-built” material properties is of the utmost importance due to the impact that it has on the subsequent application of safety assessment methods. In the case of reinforced concrete (RC) buildings, the concrete compressive strength is a material property that requires careful consideration [1] due to its inherent variability. This fact leads to the usual consideration of the concrete strength as being a random variable that has a certain (unknown) level of aleatory uncertainty [2]. This aleatory uncertainty is related to the inherent variability of the hardened concrete strength in existing structures [3] which can reach large values [4,5], often exceeding a coefficient of variation (CoV) of 20% [6]. Among other factors, this variability is associated with mix, casting and curing operations, which require a significant level of workmanship. Several studies (e.g. see [3,7,8]) have analyzed the impact of workmanship on the strength of hardened concrete and found that it can induce several types of variability depending on the structural system being analyzed. Primarily, expected variations can be associated to batch-to-batch variability, involving the randomness related mainly with the construction management and planning and with quality control. Likewise, member-to-member variability can occur due to the influence of workmanship in casting operations. Variations of the concrete strength can also be expected within each structural member due to the previously mentioned factors. Moreover, a recent study [9] also described cracking, damage and the selection of the testing positions within the length of a structural element as sources of potential variability.

In addition to the aleatory uncertainty associated with the concrete strength, epistemic uncertainty will also be generated due to the lack of knowledge associated with non-surveyed structural elements. Since survey plans only comprise tests on a

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few structural members in order to minimize the damage and the cost of inspection operations, the selection of a given set of elements to be tested instead of another will generate uncertainty. This uncertainty is even more important due to the low number of material tests that are generally carried out in existing buildings, a trend partially supported by existing norms (e.g. [10–13]). Often, standards regulating the assessment of existing buildings require a limited number of tests/inspections to be performed at each storey and for each type of primary component that is part of the building in order to obtain estimates of the mean values of the material properties. Nonetheless, as referred in [14], current building codes do not address the uncertainty level in the survey results and neglect the impact that sampling may have on the estimate of the dispersion of concrete strength (specifically on the estimate of the CoV) and on the corresponding estimate of the mean value. Therefore, controlling the epistemic uncertainty about the CoV of the concrete strength is a key component of a survey framework since it will affect the variability of the estimate (i.e. its precision), especially when it is based on a reduced number of tests. Moreover, this uncertainty is also seen to depend on the relation between the number of structural elements that are not tested during survey operations and the total number of structural elements of the population.

To control the extent of this uncertainty in survey operations and its impact on the estimate of the mean value of the concrete compressive strength in existing buildings, a method based on finite population statistics is proposed herein. The proposed approach will enable to effectively control the uncertainty in the estimates of the variability and of the mean value of the concrete strength in a population to improve their reliability. By accounting for the number of structural elements that are not tested during survey operations, the proposed method overcomes limitations of current standard methods and enables the development of more consistent survey frameworks to assess concrete strength in existing buildings.

2. Assessing statistical parameters in finite populations

In statistics, a population is said to be finite when it is possible to count all its elements. Statistical parameters characterizing these populations have specific features which are associated to finite size conditions. To evaluate the exact value of these parameters, knowledge about all the N independent elements of the population is required. If all the N elements are observed, the population mean is then:

$$\bar{x}_U = \frac{1}{N} \cdot \sum_{k=1}^{N} x_k \tag{1}$$

where U represents the population, N is the finite population size and x_k is an individual element of U. By the same principles, the variance of the population is given by:

$$S_U = \frac{1}{N-1} \cdot \sum_{k=1}^{N} (x_k - \bar{x}_U)^2 \tag{2}$$

If instead of observing all the N elements of the finite population, a sample with size n ($n < N$) is observed, estimates for $\bar{x}_U$ and S_U can be computed. Assuming a simple random sampling of n elements without replacement from an unordered population of size N, M combinations of n elements can be defined, with M being given by:

$$M = \binom{N}{n} = \frac{N!}{n!(N-n)!} \tag{3}$$

The main characteristic of finite population statistics resides in the conditional correlation between the probabilities of observing different values that is introduced by sampling. In finite populations, increasing the sample size n will affect the estimates of the statistical parameters since the observation of element x_k will affect the probability of observing the next element in the sample, i.e. x_{k+1}. This fact leads to sampling probabilities that depend on n, thus reducing the level of statistical uncertainty (that is implicit when considering a sample to represent the population) in the estimators for the statistical parameters when compared to that of infinite populations.

In a finite population with N elements, an estimate $\hat{\bar{x}}_U$ for the real mean $\bar{x}_U$ obtained using a sample with n elements is defined by:

$$\hat{\bar{x}}_U = \frac{1}{n} \cdot \sum_{k=1}^{n} x_k \tag{4}$$

The theoretical variance of the estimator $\hat{\bar{x}}_U$ obtained with a sample of n elements is defined by:

$$S(\hat{\bar{x}}_U) = \frac{1}{n} \cdot \left(\frac{N-n}{N-1}\right) \cdot S_U \tag{5}$$

where $\left(\frac{N-n}{N-1}\right)$ is the squared value of the finite population correction factor [15]. Based on Eq. (5), the variance of the estimate of the mean can be seen to converge to zero as n converges to N, which implies that the sample mean will converge to the true population mean at a rate given by the finite population correction factor. Therefore, this factor is seen as a representation of the statistical uncertainty in the estimate for the finite population mean. Still, in a general case where $n < N$, the variance of the estimate of the mean will be a direct function of S_U, thus showing the importance of knowing the variability of the concrete strength in order to control the uncertainty in the estimate of the mean. However, since the population variance S_U is always unknown, it needs to be replaced by its estimator $\hat{S}_U$ which, for a finite population, is given by [15]:

$$\hat{S}_U = \frac{1}{n} \cdot \frac{N}{N-1} \cdot \sum_{k=1}^{n} (x_k - \hat{\bar{x}}_U)^2 \tag{6}$$

The variance of the estimator $\hat{S}_U$ depends on the selected sample (i.e. on the values x_k of the n elements observed) and is given by [15]:

$$S(\hat{S}_U) = \left(\frac{N}{N-1}\right)^2 \cdot \left(\frac{1-(n/N)}{n}\right) \cdot \frac{1}{n-1} \cdot \sum_{k=1}^{n} \left[(x_k - \hat{\bar{x}}_U)^2 - \frac{1}{n} \cdot \sum_{k=1}^{n} (x_k - \hat{\bar{x}}_U)^2\right]^2 \tag{7}$$

3. Using finite population statistics to assess concrete strength in existing RC buildings

3.1. Discretizing the concrete strength and disaggregating its variability

By depending on both n and N, finite population statistics enable to control the epistemic uncertainty about the estimates of the mean and of the variability of a population using data provided by a ratio of n/N elements. This approach is somehow similar to the uncertainty reduction principle that underlines the procedures in current standards (e.g. see [10]) where it is implicit that an increase in the number of structural elements that are tested during survey operations will lead to a reduction of the uncertainty about the estimate of the mean value of the material property. Therefore, a procedure based on finite population statistics like the one proposed herein is found to be consistent with current standard assessment procedures.