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Empirical estimation of the load bearing capacity of masonry walls under buckling – Critical remarks and a new proposal for the Eurocode 6

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HIGHLIGHTS

- Reviewing the state-of-art of empirical methods for masonry buckling.
- Studying the influence of material parameters on the load bearing capacity.
- Critical remarks have been made on the state-of-art empirical methods.
- A method has been proposed to overcome the existing problems.
- A new formula for the Eurocode 6 has been proposed.

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ABSTRACT

Several empirical formulae were proposed for the practical estimation of the load bearing capacity of masonry walls subjected to concentric or eccentric vertical loading taking into account the slenderness of the wall. In the current contribution, critical remarks were made on the limitations and inconsistency of current empirical methods. The accuracy, shortcomings, and plausibility of the current existing empirical formulae have been intensively studied and compared with a reference numerical solution. It has been found that many of the available empirical methods may give unrealistic results, which lead to an overly conservative design. This can be seen particularly in the regression model-based formulae, which are less based on rational principles like the one in Eurocode 6. To solve this problem, the concept of equivalent elastic modulus has been introduced and a new empirical method has been derived taking into account the nonlinearity of the material. This method gives accurate results in comparison with the reference numerical solution and can fully exploit the considerable reserves of material strength. The new empirical method has been used as a basis to develop a formula for the Eurocode 6. The proposed formula has been showing good fitting with the experimental data.

1. Introduction

In the last decades, several numerical and analytical solutions have been developed for determination the load bearing capacity of vertically loaded masonry walls (Bakeer [1]). These methods, because of the complex mathematical formulation, are improper in practice and standards even though the results can be represented graphically. Attempts have been made to simplify the theoretical solutions into cumbersome formulae which demonstrate good agreement with accurate results. Most of the proposed empirical formulae are based on the regression models with parameters determined at the best-fit with the accurate solution, e.g. the Gaussian bell-shaped function in EN 1996-1 proposed by Kirtschig [2], or the formulae proposed by Sandoval/Roca [3]. However, the regression models may lead to wrong or unrealistic results if applied out-of-range of the data used for that approximation.

In the original draft of Eurocode 6, the capacity reduction factor has been approximated by a linear formula, but this has been criticized because the formula gives rise to negative capacity reduction factors at high values of slenderness and replaced later on in ENV with the exponential formula of Kirtschig (Hendry [4]). However, critical remarks also have been made on the empirical formula of Eurocode 6 as well, because it doesn't consider softer types of masonry in some European countries e.g. Denmark (Bakeer et al. [5]).

The present contribution aims at providing solutions to overcome the problems of the current empirical methods. An intensive study of the current state of empirical methods has been made and

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Nomenclature

c	material parameter represents the degree of nonlinearity and is equal to the ratio of the ultimate strain to the elastic strain at failure stress $= E \cdot \varepsilon_c / f = K_E \cdot \varepsilon_c$;
c_a	approximate equivalent value of the material parameter c for nonlinear material model;
C_m	correction factor for the moment magnifier method takes into account the end conditions and moment distribution in the element;
e	total eccentricity in the cross section considering both, the first order and second order eccentricities;
e_0	eccentricity at the top end of the wall;
e_{cr}	total eccentricity at the transition cross-section;
e_m	total eccentricity at the mid-height of the wall;
E	initial modulus of elasticity of masonry;
E_σ	tangent modulus of elasticity at stress level σ;
$E_{1/3}$	secant modulus of elasticity from the mean of the strains of all measuring positions occurring at a stress equal to $f/3$;
E_{eq}	equivalent elastic modulus considering the nonlinearity in the material $= \gamma \cdot E$;
f	compressive strength of masonry;
f_k	characteristic compressive strength of masonry;
f_t	tensile strength of masonry;
h	clear height of a masonry wall or column;
K_E	ratio of initial elastic modulus to compressive strength of masonry $= E/f$;
L	moment magnifier factor;
m_ϵ	second material integral defined in Eq. (89);
M	bending moment at the middle of the element;
M_0	initial bending moment at the middle of the element;
n	material parameter;
n_a	equivalent approximation of the material integral n_ϵ;
n_ϵ	first material integral defined in Eq. (8);
N	compressive load;
N_e	material failure load;
N_s	stability failure load;
r_e	relative total eccentricity in the cross section
r_{e_0}	relative eccentricity of the compressive load acting on the top end of the wall or vertical structure member $= e_0/t$;
$r_{e_{cr}}$	relative total eccentricity at the transition cross-section $= e_{cr}/t$;
r_{e_m}	relative total eccentricity at the mid-height of the wall $= e_m/t$;
r_h	relative height of masonry wall or column $= h/t$;
t	thickness of a masonry wall or column;
α	ratio of the equivalent elastic modulus for a nonlinear material model to the initial elastic modulus $= E_{eq}/E$;
δ	relative deflection at the middle of the compression element $= \Delta/t$;
δ_0	relative initial eccentricity at middle of compression element $= \Delta_0/t$;
Δ	deflection at middle of compression element;
Δ_0	initial deflection at middle of compression element;
ε	strain;
ε_c	strain corresponding to the compressive strength of masonry $= c/K_E$;
ε_t	ultimate tensile strain of masonry;
ε_u	ultimate compressive strain of masonry;
$\bar{\varepsilon}$	relative/normalized strain taken with respect to the compressive strain ε_c at pick $= \varepsilon/\varepsilon_c$;
ϵ	relative/normalized maximum strain at one edge of a cross section $= \varepsilon_1/\varepsilon_c$;
ϵ_t	relative/normalized ultimate tensile strain of masonry $= \varepsilon_t/\varepsilon_c$;
ϵ_u	relative/normalized ultimate compressive strain of masonry $= \varepsilon_u/\varepsilon_c$;
λ	slenderness ratio $= r_h/\sqrt{K_E}$;
λ_t	slenderness ratio at which the material and stability failures occur at the same time;
σ	stress;
$\bar{\sigma}$	relative /normalized stress taken with respect to the compressive strength $= \sigma/f$;
Φ	capacity reduction factor due to eccentricity and slenderness;
Φ_e	capacity reduction factor due to eccentricity;
Φ_E	capacity reduction factor of Euler's load $= \pi^2/(12\lambda^2)$;
Φ_s	capacity reduction factor associated with the stability failure load;
Φ_{cr}	corresponding reduction factor at $r_{e_{cr}}$

the outcome of the evaluation has been used to propose a new solution.

In this study, the considered masonry wall is hinged at both ends and subjected to equally eccentric or concentric loading at the top and bottom of the wall (Fig. 1).

Since masonry is a material with low tensile strength, the wall may crack under certain conditions leading to further complications due to the reduction in the effective cross-section. Masonry members under compression may fail either because of material over-stressing for squat members or because of stability failure of slender members. For squat masonry members, the failure takes place if the compressive strain at any cross-section reached the ultimate compressive strain of the material. Nevertheless, for slender masonry elements the failure occurs before reaching the ultimate compressive strain of the material at any cross-section. The former mode of failure called **material failure** and the later one called **stability failure**.

It is more appropriate for the presentation of the empirical formulae to use a relative form description for the buckling problem of masonry walls. For practical use and standards, the load bearing capacity is represented by the **capacity reduction factor** Φ for the compressive strength allowing the actual conditions:

$$\Phi = \frac{N}{f \cdot t} \tag{1}$$

where f is the compressive strength of masonry, N is the vertical load bearing capacity per unit length, and t is the thickness of the wall.

The capacity reduction factor Φ is influenced by the relative eccentricity of the load applied at the ends of the wall $r_{e_0} = e_0/t$ and the slenderness ratio of the wall λ which, in turn, depends on the geometry of the wall and the boundary conditions. The slenderness ratio λ is defined as follows:

$$\lambda = \frac{r_h}{\sqrt{K_E}} \tag{2}$$

where $r_h = h/t$ is the ratio of the height of the wall h to its thickness t, $K_E = E/f$ is the ratio of the initial elastic modulus E to the compressive strength f of masonry material.