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# Three-dimensional numerical modeling and simulation of the thermal properties of foamed concrete

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## HIGHLIGHTS

- A random generation method was extended from 2D to 3D.
- Finite volume method was used to solve the heat transfer equations.
- 3D effects on the effective thermal conductivity predictions were discussed.

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## ABSTRACT

In this paper, a three-dimensional method was developed for modeling the heat transfer of foamed concretes with a large range of densities (300–1700 kg/m$^3$). A random generation method was extended from two dimensions (2D) to three dimensions (3D) for reproducing the microstructure of foamed concrete. A finite volume method (FVM) was then used to solve the energy transport equations for two phase coupled heat transfer through the porous structure. The effective thermal conductivities (ETCs) of foamed concretes were thus numerically calculated and the 3D predictions were compared with the existing experimental data and other analytical models. The numerical results show that the predicted effective thermal conductivity varies with the lattice number in the third dimension following an exponential relationship, and it needs at least 20 lattices along the third dimension to stabilize the simulation results. In addition, the 3D numerical predictions agree more with the experimental results, since the heat conduction in the third direction is omitted in 2D simulation, leading to the underestimation of effective thermal conductivities prediction in the same boundary conditions. Finally, a correlation was then derived between the results computed with 3D and 2D numerical models.

## 1. Introduction

Nowadays, various kinds of lightweight concretes have been widely used in construction industry [1–4]. The interest is of course to decrease the amount of load-bearing elements, as well as to get better thermal properties compared with conventional concrete. The latter point especially makes sense with regard to relieve the energy crisis and ecological problems in developing countries, where the large-scale construction of urbanization is proceeding. Lightweight concrete can normally be produced by replacing totally or partially the standard aggregate with low weight and usually low cost components (e.g., EPS, perlite and clay), or by directly introducing gas or foam into the cement or mortar paste, forming the so called foamed concrete.

Foamed concrete has been widely used in non-structural applications [5–8], mostly for cast-in-place applications in roof slopes, floor leveling and insulating layers of wall constructions, and for any kind of void filling projects (mines, tunnels, road basements, ground stabilization and others), owing to its self-compacting, light weight, excellent thermal insulation properties and affordable strength values. In many application cases, there are no strict requirements to strength characteristics, and thermal conductivity plays the more dominant role [9–12]. In addition, it is possible to design the properties of foamed concrete to meet the construction requirement by varying material parameters such as cement paste composition, foam size and volume friction. To this end an accurate evaluation of the relationship between the microstructure and thermal transfer properties of such porous lightweight building materials is required.

At ambient temperature, the heat transfer in solid foams is dominated by thermal conduction. Numerous of authors proposed empirical, analytical or numerical model depending on the porous

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## Notation

| | |
|---|---|
| $A$ | fitting parameter in Eq. (7) |
| $B$ | fitting parameter in Eq. (7) |
| $c$ | probability of a point in a given space to become a center of the bubble, (0,1) |
| $f_{(X,\mu,\sigma)}$ | log-normal distribution function |
| $k$ | thermal conductivity ($\mathrm{W\,m^{-1}\,K^{-1}}$) |
| $k_{3D}$ | 3D numerical prediction of thermal conductivity ($\mathrm{W\,m^{-1}\,K^{-1}}$) |
| $k_{2D}$ | 2D numerical prediction of thermal conductivity ($\mathrm{W\,m^{-1}\,K^{-1}}$) |
| $k_{eff}$ | effective thermal conductivity ($\mathrm{W\,m^{-1}\,K^{-1}}$) |
| $k_f$ | thermal conductivity of fluid phase ($\mathrm{W\,m^{-1}\,K^{-1}}$) |
| $k_g$ | thermal conductivity of air ($\mathrm{W\,m^{-1}\,K^{-1}}$) |
| $k_s$ | thermal conductivity of solid phase ($\mathrm{W\,m^{-1}\,K^{-1}}$) |
| $L$ | number of lattices along $x$-axis |
| $M$ | number of lattices along $y$-axis |
| $N$ | number of lattices along $z$-axis |
| $P$ | number of lattices with unknown temperature |
| $Q_k$ | heat fluxes along the $y$-axis (W) |
| $R$ | number of target bubbles |
| $T$ | temperature (K) |
| $T_{cold}$ | temperature of cold boundaries (K) |
| $T_{hot}$ | temperature of hot boundaries (K) |
| $X$ | pore diameter, mm |
| $x$ | coordinate along the $x$-axis |
| $y$ | coordinate along the $y$-axis |
| $z$ | coordinate along the $z$-axis |

*Greek letters*

| | |
|---|---|
| $\delta$ | overlap ratio |
| $\varepsilon$ | porosity |
| $\mu$ | mean value of variable's logarithm |
| $\sigma$ | deviation of the variable's logarithm |

morphology and on the conductivity of the phases to estimate the effective thermal conductivity (ETC) of this type of materials.

Some analytical models which are often used to describe the effect of pore volume fraction as a variable on the thermal conductivity of a porous material are summarized in Table 1. In each case, the expression is based on a geometrical simplification of the microstructure concerning the spatial distribution of the two phase system. For example, the Hashin and Shtrikman [14] expressions give the most restrictive upper and lower limits of the effective thermal conductivity (ETC) for a two-phase system where spherical inclusions are placed in a continuous matrix. Landauer [15] derived a practical expression in which the connectivity of the phases is taken into account. This approach is also called "Effective medium percolation theory" (EMPT). These approaches become limited when the pore volume fraction increases and the isolated pores become connected. Recently, owing to the rapid developments in computer and computational techniques, some numerical models have also been used to predict the thermal conductivity of porous materials [16–21]. Coquard and Baillis [20] calculated the ETC of two-phase heterogeneous materials by using a numerical finite volume method. The work of Wang and Pan [21] who solved the energy transport equation through random open-cell porous foams using a high-efficiency Lattice Boltzmann method can also be cited. Wei et al. [22] presented a random method to predict the effective thermal conductivity of foamed concrete, which included a random generation method to reproduce microstructure of foamed concrete and a resistor network analogy method to solve the energy transport equations for fluid–solid coupled heat transfer. Wei's predictions agreed well with the experimental data when the porosity is less than 35%. However, this kind of 2D models underestimated the predictions of ETC in high porosity cases, attributed to the inherent approximations in the 2D analysis.

Based on the previous work [22], this paper developed a three-dimensional random generation method to reproduce the 3D microstructure of foamed concretes. A finite volume method (FVM) was also developed which allows the effective thermal conductivity of foamed concretes with flexible microstructure to be predicted numerically. Finally, the proposed model was validated by the comparison with the two-dimensional predictions, the experimental data and other existing models. The 3D effect of prediction of ETC of foamed concrete was therefore discussed.

## 2. Numerical methods

### 2.1. Random generation of 3D porous structure

As mentioned in the introduction, the existing analysis models are based on the geometrical simplification of the microstructure. In such models, the stochastic natures of the porous material are neglected. To bring the random characteristics of porous materials into modeling, the random effect has to be introduced during the generation of porous media structure. The random location of lattices is the most easy and popular method to construct an artificial porous medium [16,23,24]. However, this method cannot reflect the real microstructure of the studied porous material. Wang et al. [25] also proposed a model to reproduce micro-morphology of random porous media based on the cluster growth theory. Recently, Wei et al. [22] presented a full description of the generation-growth method for foamed concrete with the pores forming randomly using some geometric characteristics which were estimated from 3D tomographic images. Numerical simulation in these model foams allows an investigation of relations between the geometric structure of the porous material and its thermal transfer properties. This method has been used in two-dimensional (2D) simulation and has shown good agreement with a series of experimental results. Here we implement this method in three-dimensional (3D) isotropic system. In previous work [22], the microstructures of foamed concretes were characterized by 3D-XCT and they found that the bubble size distribution follow a log-normal relationship as follows:

$$f(X;\mu,\sigma)=\frac{1}{x\sigma\sqrt{2\pi}}e^{\frac{(\ln X-\mu)^2}{2\sigma^2}},\quad X>0 \tag{1}$$

where $\mu$ and $\sigma$ are the mean and standard deviation of the variable's logarithm, respectively.

For the foamed concretes studied in this paper, the same assumption for bubble size distribution mode in 3D structure was made and the corresponding fitting values of $\mu$ and $\sigma$ are also summarized in Table 2.

The generation process for such porous structures of foamed concretes is described as follows:

(1) Randomly locate the pore centers based on a uniform distribution probability, $c$, which is defined as the probability of a point in a given space to become a center of the bubble.