Construction and Building Materials 36 (2012) 665–673

Contents lists available at SciVerse ScienceDirect

Construction and Building Materials

journal homepage: www.elsevier.com/locate/conbuildmat

Prediction of elastic modulus of normal and high strength concrete using ANFIS and optimal nonlinear regression models

Behrouz Ahmadi-Nedushan *

Civil Engineering Department, Engineering Faculty, Yazd University, Pajoohesh Street, Safa-ieh, Yazd, Safa-ieh, P.O. Box 89195-741, Yazd, Iran

HIGHLIGHTS

- ▶ A general nonlinear regression model is presented and optimized.
- ▶ ANFIS outperforms the optimal nonlinear regression models for both HSC and NSC.
- ▶ ANFIS outperforms most of other predictive models proposed in the literature.

ARTICLE INFO

Article history:
Received 28 February 2012
Received in revised form 3 May 2012
Accepted 4 June 2012
Available online 15 July 2012

Keywords:
ANFIS
Regression
Concrete
Compressive strength
Elastic modulus
High-strength concrete
Optimization

ABSTRACT

This article proposes an adaptive network-based fuzzy inference system (ANFIS) model and three optimized nonlinear regression models to predict the elastic modulus of normal and high strength concrete. The optimal values of parameters for nonlinear regression models are determined with differential evolution (DE) algorithm. The elastic modulus predicted by ANFIS and nonlinear regression models are compared with the experimental data and those from other empirical models.

Results demonstrate that the ANFIS model outperforms the nonlinear regression models and most of other predictive models proposed in the literature and therefore can be used as a reliable model for prediction of elastic modulus of normal and high strength concrete.

1. Introduction

One of the most important elastic properties of concrete is its modulus of elasticity, which is defined as the slope of the stress strain curve within the proportional limit of a material [1]. The modulus of elasticity is a measure of stiffness or the resistance of material to deformation and is used in many analysis and design calculations such as estimation of immediate and time-dependent deformations, evaluation of stiffness of structural members and estimation of creep and shrinkage in reinforced and pre-stressed concrete structures.

The measurement of elastic modulus of concrete is a relatively elaborate test procedure involving cyclic loading and multiple strain measurements [2]. As measurement of elastic modulus is more complicated and time-consuming in comparison with the measurement of compressive strength, national codes generally provide empirical formulas to relate the elastic modulus to compressive strength.

ACI building code [3] and TS 500 [4] recommend the following two equations for computing elastic modulus E_c (in GPa) from compressive strength f_c (in MPa) for normal strength concrete (NSC):

$$E_c = 4.73(f_c)^{0.5} \quad (1)$$

$$E_c = 3.25(f_c)^{0.5} + 14 \quad (2)$$

For high strength concrete (HSC), building codes propose different relationships between elastic modulus and compressive strength. For instance, American [5], European [6], and Norwegian [7] committees recommend the following equations respectively:

$$E_c = 3.32(f_c)^{0.5} + 6.9 \quad (3)$$

$$E_c = 10(f_c + 8)^{0.33} \quad (4)$$

$$E_c = 9.50(f_c)^{0.3} \quad (5)$$

* Tel.: +98 3518255073; fax: +98 3518210699.
E-mail address: Behrooz.Ahmadi@Gmail.com

0950-0618/$ - see front matter © 2012 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.conbuildmat.2012.06.002

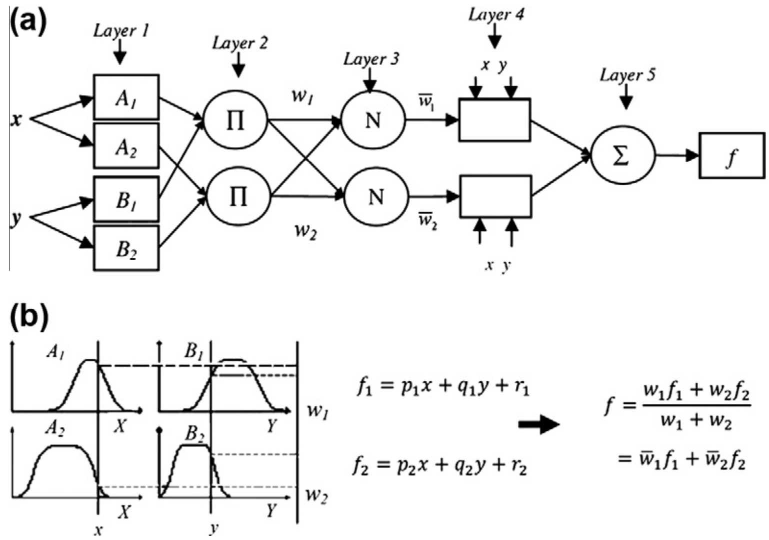

Fig. 1. (a) Architecture of ANFIS and (b) fuzzy-reasoning scheme of ANFIS [22].

In the last decade, researchers have proposed different empirical approaches to estimate the elastic modulus of NSC and HSC from its compressive strength.

Demir (2005) used fuzzy logic for prediction of the elastic modulus for NSC and HSC [8]. Demir (2008) developed three different architectures of feed forwarded artificial neural network (ANN) for prediction of the elastic modulus for NSC and HSC [9]. Yan and Shi (2010) proposed a support vector machine (SVM) model to predict the elastic modulus of both NSC and HSC data [2]. Gandomi et al. (2011) proposed different models based on linear genetic programming (LGP) for NSC and HSC. Separate equations were proposed to relate the elastic modulus of concrete to compressive strength for NSC and HSC [10].

This article investigates the suitability of two different types of models: adaptive network-based fuzzy inference system (ANFIS) and optimal nonlinear regression models for predicting the elastic modulus of concrete based on its compressive strength. ANFIS is a fuzzy based inference system, where the parameters of fuzzy systems are obtained by back propagation learning algorithm. An ANFIS model combines the advantages of ANN and fuzzy logic.

In nonlinear regression models, differential evolution (DE) is used to find optimal parameters which result in the best accuracy and the minimum errors. Comparison of results of methods developed in this article and those from other methods proposed in the literature can reveal the strengths and weaknesses of different empirical methods. Since building codes generally provide empirical functions based on the compressive strength, compressive strength is considered as the only input in the proposed models.

The optimal nonlinear regression models and ANFIS are described in details in the next sections followed by a brief description of the datasets used for the development and evaluation of the proposed models. Finally, the results of the analysis of the data and performances of the models are presented and discussed.

2. Theory and methodology

2.1. Optimal nonlinear regression models

A typical nonlinear regression model is defined as follows:

$$y = f(x,\theta) + \varepsilon \tag{6}$$

where ε represent normally distributed errors, y is response, x represents the inputs, θ are parameters and $f(x, \theta)$ is a nonlinear function of θ.

A review of Eqs. (1)–(5) proposed by building codes for NSC and HSC reveals that a general nonlinear regression model of the form:

$$E_c = a(f_c + b)^c + d \qquad b \text{ and/or } d = 0 \tag{7}$$

can be effectively used to represent all the proposed equations. In Eq. (7), parameters b and d act as intercept and therefore only one of them is needed to be in the model.

For instance, the formula provided by ACI for NSC (Eq. (1)) can be obtained by setting both b and d to zero ($E_c = a(f_c)^c$) and ACI formula for HSC (Eq. (3)) may be obtained by setting b to zero ($E_c = a(f_c)^c + d$).

The values of parameters of Eq. (7) can be determined by minimizing the error between observed and predicted values. Defining these parameters as the variables to be optimized and root mean squared errors (RMSE) as the objective function, an appropriate optimization algorithm can be used to determine the optimal values of the parameters.

In recent years, in order to deal more efficiently with optimization problems and to compensate the drawbacks of traditional nonlinear programming methods, a wide variety of evolutionary algorithms have been proposed. Evolutionary algorithms are heuristic techniques mostly inspired by natural phenomena. In this article, we used the differential evolution (DE).

2.2. Differential evolution

DE has been proven to be efficient, robust and easy to implement in finding the optimal solution of various problems [11,12]. DE optimizes a problem by maintaining a population of candidate solutions and creating new candidate solutions using the operators of mutation, crossover and selection. Each candidate solution is a vector that contains as many variables as the dimensions of the problem. The current population, symbolized by $P_{x,g}$ is composed of vectors, $X_{i,g}$, that have already been found to be acceptable either as initial points, or by comparison with other vectors [13]. The population is defined as:

$$\begin{aligned} &P_{X,g} = (X_{i,g}), \quad i = 1,2,\ldots,N_p, \quad g = 1,\ldots,g_{\max} \\ &X_{i,g} = (x_{j,i,g}), \quad j = 1,2,\ldots,D \end{aligned} \tag{8}$$

In the Eq. (8), N_p denotes the number of population vectors, the index g represents the generation counter, g_{max} is maximum number of generations and i is the population index parameters within each vector [14].