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The effect of geometry on the structural capacity of masonry arch bridges

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ABSTRACT

This paper presents the effect of geometry on the structural capacity of masonry arch bridges with different geometric features. This study was performed using an application (ANPAF) developed in MATLAB and based on the Linear Programming Method developed by Livesley. The geometry is read directly from an 'a.dxf' file, which stores the information obtained from planimetric surveying techniques. The results were compared with real and idealized geometry corresponding to each of the arches. This study aims to estimate the percentage of error that can occur in the structural assessment of masonry bridges by reading from different shapes as well as to estimate geometrical error.

1. Introduction

Masonry arch bridges date back over 4000 years. All of those bridges were built following proportional rules. The constructors and engineers in the 15th–19th centuries, and probably the ancients, used structural rules to determine the structural dimensions of this type of structures. Although Galileo Galilei (1564–1642) attempted to demonstrate the impossibility of these type of rules, it can be said that the master builders employed these type of proportional rules [1]. From this point of view, all masonry arches were designed by taking into account the geometry and proportions that ensured stability.

One of the most important geometric factors of an arch is the thickness of the thread, so since the 18th century several authors have proposed different equations to determine its value, probably based on their experience [2–5]; note that there are a lot of disparity on the values offered by these equations.

Many of the bridges that were originally built for the passage of carts are currently being used for traffic and even heavy goods vehicles in some cases. This change in use and the abandonment of many of these bridges produced a change in their geometry and as a consequence in the guideline of the arches, thus changing their stability and load capacity.

Over recent decades, great efforts have been made to better understand the structural behavior of these bridges and to ensure their structural safety, as they are now being used for purposes other than those for which they were designed [6–8]. In most of cases the geometry employed to determine the load capacity has been an ideal geometry because of the difficulties in working with the real geometry. Now, with new technology, it is possible to take the real geometry and work with it. Thus, it is necessary to study how the geometrical properties of these arch bridges are important and how they determine the structural behavior of the arches [9,10].

There are different Plastic Methods available for analyzing the stability of arches: the Heyman Method [11,12], the Virtual Works Method [13–15], the Graphical Method [16] and the Livesley's Linear Programming Method [17,18].

The Linear Programming Method was developed by Livesley in 1978 [17] as a technique to analyze rigid-plastic structural frames in order to provide a formal procedure for finding the safety load factor (λ) of any structure formed from rigid blocks. The masonry is considered as a set of independent blocks and infinitely rigid blocks that moves without deformation. Under this view, the stability of each voussoir is studied by obtaining the reactions in each voussoir face. The system resolution is performed by linear programming. This method, unlike previous ones, starts from a known load, and looks for the safety coefficient (λ) of the structure for this load. The load factor (λ) is maximized subject to the equilibrium equations of the structure and linear constraints imposed by criteria for failure at the voussoirs' interfaces. If this safety factor is less than one, the system would not be stable. The collapse load corresponds to the product of the imposed load and the safety coefficient λ.

Once factor λ is maximized and the reactions of each voussoir for the collapse load are obtained, the set of hinges is obtained on the basis of the restrictions imposed. The thrust line is taken

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out by looking for the crossing point of the resultant pressure on the voussoirs' interfaces.

In the present article the method employed is Livesley's Linear Programming Method because it permits us to take the joints and work with them. Other advantages of this method over other methods, such as the fact that this method requires less computational time to calculate the collapse load for the same arch, are described in "Comparative structural analyses of masonry bridges: An application to the Cernadela Bridge" [20].

The structural study of masonry arch bridges was performed using an application (ANPAF) [19,20,22] developed in MATLAB and based on the Linear Programming Method developed by Livesley. The ANPAF tool is divided into two blocks. The geometric block deals with reading the image or an 'a.dxf' file. The second block carries out the structural analysis, acquires the collapse mechanism and draws the thrust line using different methods.

The present study followed three steps: (1) a planimetric survey of the bridges using a total station and collected historic dates about their construction, (2) a geometric analysis of data obtained as an estimation of the arches' geometric ideal and possible starting point of construction and (3) an estimation of the structural load capacity in each case and the influence of the geometric differences.

2. Geometric data acquisition

In order to evaluate the effect of the geometry on the structural capacity of the masonry arch bridges, six bridges from different periods have been chosen, all of them in Spain. Fig. 1 shows images of these bridges. Fig. 1a shows the Traba bridge located in A Coruña, whose construction is estimated to be between the 14th and 15th centuries; it has four arches, two of which are clearly pointed arches and the other two are semi-circular arches. Fig. 1b shows the bridge in Miravete de la Sierra (Teruel), which has a segmental arch built in the 17th century. Fig. 1c is the medieval bridge of Pontevea, located between Pontevedra and A Coruña. It was built in the 15th century, has six eyes – three of them are middle point arches, and the other three have a slightly pointed line. Fig. 1d shows the As Partidas Bridge, which is possibly of Roman origin and has undergone severe transformations over time; it consists of six arches with different shapes: four are round and two are pointed. Fig. 1e is the Puntaleiras Bridge, located in As Neves (Pontevedra). It is a medieval bridge from the 12th century with one only middle point guideline arch with several deformations. And Fig. 1f shows the sixth bridge, a medieval bridge located on the river Louro in Pontevedra. It has four arches: two middle point arches and two pointed ones; one of the arches has large deformations and stone slides.

The set of all bridges makes for a study of 22 arches. However, one of them was ruled out: the smallest arch of the Traba Bridge with a span of 3.19 m. Its study requires a separate analysis because the arch is unstable under its deadweight when taking into account only the geometry. Therefore, the present study is of 21 arches: 11 middle point arches and 10 pointed. The values of the arch spans (s) are between 1.72 m (As Partidas Bridge) and 13.33 m (As Partidas Bridge), and the thickness (t) of the arches is between 0.30 m (Miravete de la Sierra) and 0.81 m (As Partidas Bridge). The value of the thickness (t), considered to be one of the most important parameters, was obtained as a fraction of the span. In the 15th century the first empirical rule for calculating thickness was proposed as $t = s/10$, measured at the key of the arch [9]. This rule, however, often did not produce accurate measurements. Later, between the 18th and 20th centuries, some authors looked at this relationship and gave different empirical rules to obtain the minimum thickness (t) value for the stability of the arch. The empirical rules from those centuries are collected in Table 1.

Other authors have studied the limit values of arch thickness (t) to function under its own weight, both for middle arches or pointed arches [1]. The first scientific study was conducted by Couplet in 1730, establishing a relationship $k = t/R = 1/10$ (R being the average radius of the arch, and k the minimum depth for the arch under its deadweight). Later in 1802, Rondelet, based on testing, set a lower thickness: $k = 1/18 = 0.1053$. Audoy (1820) repeated previous studies, Petit (1835) established $k = 0.1078$, Méry (1840) graphically deduced the value $k = 0.1079$, and Heyman (1969) set $k = 0.1060$. More stringent values were obtained in 1907 by Milankovitch, where $k = 0.1075$; this was corroborated by Ochsendorf in 2002 [21].

Table 2 gathers the geometric characteristics of the arches studied in this paper. Basic geometric data are the span (s), rise (r) and thickness (t). The arches have been duly sorted from lowest to highest span value. At the same time, each arch has been associated with the empirical rule that its value is closest to the value of the thickness and the relationship $k = t/R$. The other columns illustrate the relationship r/s, t/r and t/s. The relationship r/s indicates whether the arch is a segmental arch, a semi-circular arch or a pointed arch. The values that range between $0.5 \geqslant r/s > 0.4$ are considered semi-circular arches, values of $r/s \leqslant 0.4$ are segmental arches and, if $r/s > 0.5$ an arch is considered pointed [9]. Fig. 2 shows the relationship between (a) r–s, (b) r/s–s, (c) t/r–s for the different arches studied. The relationship t/r indicates whether the thickness of the arch is greater than the thickness limit proposed by the authors mentioned above.

3. Geometric analysis

Currently there are a great variety of techniques that allow planimetric documentation of built heritage, both for buildings and for masonry arch bridges. All planimetric surveys require two phases: data collection and data interpretation using different techniques. Because planimetric surveys require different instrumentation and methodology, data collection should always be accompanied by a series of drawings or sketches made on site. Methods used in data collection are: (1) traditional methods: tape measure, laser meter, plumb, level, etc.; (2) topographic methods: total station with laser meter (Transaction Processing System – TPS); (3) photogrammetric methods: digital cameras and metric cameras; and (4) Terrestrial Laser Scanner (TLS) methods: 3D scanner. Depending on the method chosen, data collection reaches a precision between 5 cm and 6 mm [22,23]. In some cases these methods can be complementary.

The planimetric survey of the bridges studied in this paper was performed using TPS. The equipment for TPS consists of the new generation surveying instrument, a total station, which integrates the electronic measurements of distance and angles on a single computer, and internal communications that allow the transfer of data to an internal or external processor. TPS is capable of measurement multitasking, saving data in real time. Fieldwork is simple and orderly. Through proper planning of fieldwork, the entire bridge is covered. Each new position of the TPS is estimated from the previous position, the first being the reference taken. At the end, it makes a calculation error of closure and compensation for calculating angles and azimuths.

There are three ways to take the geometry: (1) obtaining the coordinates that define the guideline of the arch, (2) obtaining the coordinates that define the voussoir contacts and (3) taking the main measurements like span, rise and thickness and idealizing the geometry.

In the first case, with the information obtained with the TPS and complemented by a set of photographs, a 3D wireframe model is stored as an 'a.dxf' file, from which the plans and elevations are ob-