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Comparative assessment of load–resistance factor design of FRP-reinforced cross sections

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ABSTRACT

In the last decades, codes have implemented the load–resistance factor design (LRFD) approach to achieve a certain safety level in structural sections. Recently, the same philosophy established in the case of steel bars was adapted for reinforcement by innovative materials such as fiber-reinforced polymers (FRPs). LRFD is claimed to be a semi-probabilistic approach, although the implied safety is not intelligible by practitioners, being hidden into the so-called safety factors (SFs) prescribed by codes, which should account for load- and strength-affecting heterogeneities. Often, especially in the case of FRP reinforcement, the SFs differ from one code to another because of the format of the design equations. The objective of the simple study presented in the paper is to compare the safety levels, expressed in terms of conventional probability of failure, for different codes at the state-of-the art with respect to the design of FRP-reinforced concrete worldwide. The purpose is to investigate how the different equation formats, design values of material properties, and partial safety factors, affect the implicit design safety and whether it is similar among international guidelines. The study considers design of cross sections in bending at the ultimate limit state according to: ACI 440.1R-06 (US guidelines), CAN/CSA-S806-02 (Canadian guidelines), and CNR-DT 203/2006 (Italian guidelines, for which sensitivity of design to SFs is also investigated). For comparison purposes, design of steel-reinforced sections is considered according to the recent Italian regulations. Results indicate that reliability indices achieved with design procedures are generally comparable among the considered codes, and larger than that referring to steel reinforcement.

1. Introduction

This section briefly reviews the basics of safety formats in international design codes and motivates the study. In fact, although the most of the background may be found in well-known literature (e.g., [1]), it may be worthwhile to recall the working hypotheses of current practice, as they are not directly intelligible from guidelines. At the end of the introduction, the framework of the guidelines is presented and the organization of the work is given.

The objective of structural design is that the construction warrants a given safety margin with respect to some feared *failure mode*. In fact, structural safety has to refer to an undesired condition (*limit state* hereafter), which may lead to some unacceptable situation, namely *failure*. The quantification of safety consists of the *reliability* assessment, that is, the evaluation of the *probability of safe behavior*, P_s. For any engineering system P_s has to be referred to the time in which it operates; e.g., the *design life* (T).

Structural reliability has to be necessarily expressed in probabilistic terms because most, if not all, the factors possibly determining failure are uncertain despite the values assumed in design; e.g., mechanical models, members' geometry, materials' properties, and loads. In fact, these are called random variables (RVs), $\overline{X} = \{X_1, X_2, \ldots, X_n\}$, whose actual heterogeneity is characterized by appropriate probability density functions (PDFs) for each instant in the lifetime of the structure generating, in fact, *stochastic processes* (e.g., [2]).

If the failure for the structure of interest may be expressed by a function, G, which is positive if the system is in safe conditions and is non-positive if limit state of interest is reached, for example, in the *stress–strength* model, the difference between the *resistance* (R) and its counterparts due to *loads* (L), the structural reliability may be expressed as the probability that the limit state function remains positive in the $(0, T)$ interval (being 0 the life's start time), Eq. (1), from which the probability of failure[1] (P_f) emerges.

$$\begin{aligned} P_s(T) = 1 - P_f(T) &= \Pr[G(\overline{X}, t) > 0 \; \forall t \in (0, T)] \\ &= \Pr[R(\overline{X}, t) - L(\overline{X}, t) > 0 \; \forall \, t \in (0, T)] \end{aligned} \tag{1}$$

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[1] Because, it is expected the reliability of structures to be high (e.g., P_s is relatively close to 1) may be handy to work in terms of P_f expressed as a power of ten.

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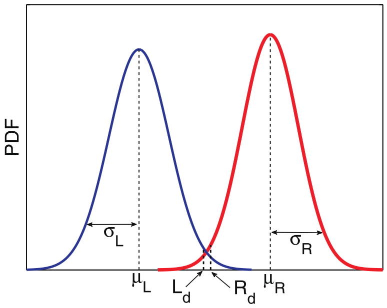

Fig. 1. *Stress–strength* model for reliability assessment. For simplicity Gaussian PDF shape is considered for both PDFs of *R* and *L*.

Calibrating the RVs in a way that their variability does not explicitly depend on time anymore (i.e., rendering the safety assessment a *time-invariant* problem; e.g., [3]) the failure probability is expressed by the integral of the joint PDF of the variables, $f_{\overline{X}}(x_1, x_2, \ldots, x_n)$, over the domain in which the set of $\overline{X}$ renders G non positive (i.e., the *failure domain* or F), as follows:

$$P_f = \Pr[\overline{X} \in F] = \int_{F:G\leqslant 0} f_{\overline{X}}(x_1, x_2, \ldots, x_n) \cdot dx_1 \cdot dx_2 \ldots dx_n \qquad (2)$$

A reliability-oriented structural code should ask the practitioner to assess the safety of a structure (either of new design or existing) computing the probability of failure and verifying whether it does not exceed some upper bound that is considered acceptable (P_f^*), as follows:

$$P_f \leqslant P_f^* \qquad (3)$$

Modern codes do not allow for such an explicit approach for various reasons, mostly related to the difficulty of giving practice-ready procedures to assess the probability of failure, and the persistent need to have a prescriptive format of design rules. In fact, Eq. (3) is replaced by one of the type of Eq. (4), which basically is a comparison of *design* values of actions due to loads (L_d) and resistance (R_d) computed via deterministic equations, which are familiar to engineers.

$$L_d \leqslant R_d \qquad (4)$$

This is done at a sectional level, while it should be more correctly computed for the whole structure; however, that would imply significant complications. Leaving the probabilistic approach to a sectional level inevitably renders the failure probabilities conventional, in a way that they do not represent failure probability of structures where such sections are employed and usable for comparison purposes only [4].

If the terms in Eq. (4) are calibrated based on the PDFs of L and R (separately if stochastically independent) this approach is considered *semi-probabilistic* and referred to as *load–resistance factor design* (LFRD); [5]. In fact, L_d and R_d reflect the probabilistic nature of L and R through some coefficients called safety factors (SFs) and applied, depending on the code, to statistics of uncertain design variables affecting R and L, or directly on measures of resistance and loads effects acting on the structural element (to follow).

When postulated, about four decades ago, LFRD was conceived to be *temporary*. It was supposed to be shortly replaced by codes allowing professionals to compute Eq. (2) explicitly for their structures [6]. Nevertheless, it is still used around the world and also adopted by regulations concerning new technologies in civil engineering, such as reinforced concrete (RC) structures employing fiber reinforced polymers (FRPs). This is mainly because its afore mentioned probabilistic basis and simplicity of application by reliability non-experts. However, codes seldom clearly report the calibration of the design parameters and the underlying hypotheses, and, therefore, safety implied in design is not directly intelligible. Moreover, design equations are custom for each code and it is also not possible to compare them in terms of implicit safety. This motivated other work (e.g., [7–9]) and the simple investigation presented in the following, where a probabilistic assessment for international codes dealing with FRP-RC cross sections is carried out. In fact, the purposes of this study may be summarized as: (i) to understand how the format of design equations, material properties assumed for computations, and safety factors (eventually partial), affect safety of cross sections at ultimate limit state in each guideline; and (ii) to address whether the different declensions of LRFD in each code imply comparable safety.

In particular, reliability analysis of flexural capacity of glass fiber-reinforced polymer (GFRP) RC cross sections at *ultimate limit state* (ULS) is performed. The analysis considers design of cross sections in bending according to three codes: ACI 440.1R-06 (*US guidelines*; [10]), CAN/CSA-S806-02 (*Canadian guidelines*; [11]), and the *Italian guidelines* CNR-DT 203/2006 [12]. For comparison, design of steel-reinforced sections according to the recent new Italian Building Code or NIBC [13] is also included. These three codes were chosen to cover the majority of modern standards concerning design of concrete reinforced with composite materials.

Case studies concern design of cross sections, according to the codes, for different values of the safety factors. Subsequently, conventional failure probability is computed and compared. Finally, for CNR-DT 203/2006, it is analyzed how sensitive is design to different SF values.

2. Basics of load–resistance factor design

Typical categories of uncertainty in structural analysis are loads, material strengths, member geometries, and there is also some uncertainty related to the mechanical (analytical) models assumed (e.g., [1,14]). If a cross section in bending is considered, uncertainty

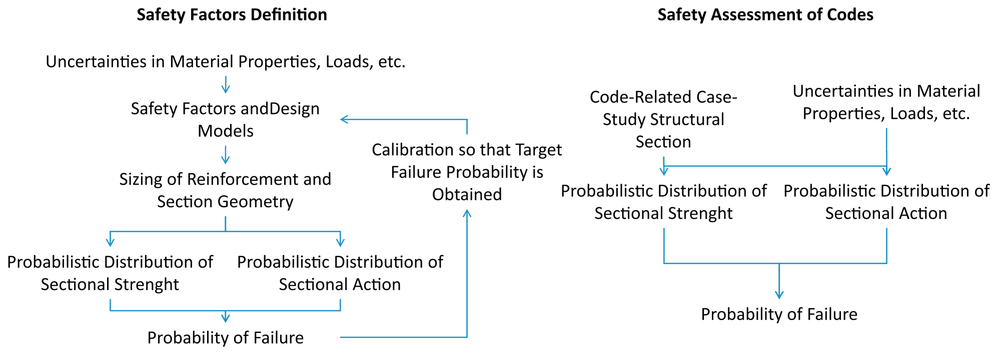

Fig. 2. Calibration of safety factors (left), and safety assessment procedure (right).