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Theoretical modelling of bowing in cracked marble slabs under cyclic thermal loading

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ABSTRACT

In the first part of the present paper, a theoretical model to calculate the progressive bowing of marble slabs submitted to thermal cycles is presented. The model, developed within the framework of linear elastic fracture mechanics, takes into account the mechanical characteristics of the marble as well as the actual cyclic temperature field in the material. The slabs are subjected to a thermal gradient along their thickness (due to different values of temperature between the outer and inner sides of the slab) as well as to thermal fluctuation on the two sides of the slab due to daily and seasonal temperature excursions. This thermal action causes a stress field which can locally determine microcracks due to decohesion of grains. Stress intensification near the cracks occurs and leads to crack propagation in the slab. Such crack propagation under thermal actions is evaluated, and, considering different crack densities in the material and different boundary conditions in the slab, the corresponding deflection (bowing), which account for both elastic bulk deformation of the slab and for the deformation due to the crack presence, is calculated. In the second part of the present paper, the model is applied to the case study of the Pescara (Italy) court building.

1. Introduction

Marble slabs are frequently used as façade panels to externally cover buildings. In some cases a bowing of such façade panels after a short time of environmental exposure is experienced. The bowing is generally accompanied by a reduction of strength properties which increases with increasing degree of bowing. In order to understand the phenomenon of bowing in marble slabs, several experimental [1] and theoretical [2–9] studies have been carried out. The results of these studies show that the strength of the rock after environmental exposition decreases due to rock damage and grain decohesion. In particular, Royer [5] showed that thermal action produces self-equilibrated stress states at calcite grain interfaces which are responsible of progressive damage in the material leading to initiation and propagation of intergranular cracks.

In situ measurements using a bow-meter [10,11] showed that the bowing of marble slabs, ranging from concave to convex shapes, is mainly dependent on the microstructure of the marble, the slab position, as well as on the fluctuation of temperature and moisture content. When thermal actions are cyclic, due for instance to daily temperature excursion, oscillating values of stress/strain develop in the material. The accumulation in an irreversible manner of the cyclic stresses/strains yields a progressive decohesion of the calcite grains. SEM analysis of calcite grains in marble slabs under cyclic loading [12] clearly demonstrated this progressive decohesion.

The determination of the overall mechanical behaviour of marble slabs on the basis of the aforementioned micromechanical phenomena might be performed within the framework of linear elastic fracture mechanics (e.g. see Ref. [13]). Accordingly, stress/strain state induced by cyclic thermal loading acting on the marble slab can be determined along with the deflection (bowing) of the slab due to both elastic bulk deformation and intergranular cracks. As the cracks propagate under cyclic thermal loading, the level of bowing after a certain number of thermal cycles and the fatigue life (expressed in terms of number of thermal cycles causing the collapse of the slab) can be calculated.

In the first part of the present paper, a theoretical model to calculate the progressive bowing of marble slabs submitted to thermal cycles is presented. The model, developed within the framework of linear elastic fracture mechanics, takes into account the mechanical characteristics of the marble as well as the actual cyclic temperature field in the material. The slabs are subjected to a thermal gradient along their thickness (due to different values of temperature between the outer and inner sides of the slab) as well as to thermal fluctuation on the two sides of the slab due to daily and seasonal temperature excursions. This thermal action causes a stress field which can locally determine microcracks due

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Nomenclature

a	crack length	t	time
C	material parameter of the Paris law	T	temperature
E	Young modulus	x	transversal coordinate of marble slab (x=0 corresponds to inner side)
f	deflection (bowing) of marble slab at mid-span section	z	longitudinal coordinate of marble slab (z=0 corresponds to mid-span section)
h	thickness of marble slab	α	thermal expansion coefficient
k	diffusion coefficients	ε_x	transversal normal strain
K_I	Mode I stress intensity factor (SIF)	ε_z	longitudinal normal strain
K_{IC}	fracture toughness or Mode I critical stress intensity factor	ε_{xz}	shear strain
L	length (span) of marble slab	ν	Poisson ratio
m	material parameter of the Paris law	σ_x	transversal normal stress
n	crack density parameter	σ_z	longitudinal normal stress
N	number of thermal cycles	τ_{xz}	shear stress
S_s	specific surface of grains		

to decohesion of grains. Stress intensification near the cracks occurs and leads to crack propagation in the slab. Such crack propagation under thermal actions is evaluated, and, considering different crack densities in the material and different boundary conditions in the slab, the corresponding deflection (bowing), which account for both elastic bulk deformation of the slab and for the deformation due to the crack presence, is calculated. In the second part of the present paper, the model is applied to the case study of the Pescara (Italy) court building.

2. Description of the theoretical model

According to linear elastic fracture mechanics, instable propagation of a crack occurs when the stress intensity factor (SIF) K_I at the crack tip reaches a limit value named fracture toughness or critical intensity factor (K_{IC}) (e.g. see Ref. [14]). However, the technical literature reports cases where cracks propagate in the marble slab for SIF values smaller than the critical one (see the cases of the Amoco Building in Chicago [15] and of the East Asian Bank in Hong Kong [13]): this phenomenon is known as subcritical cracking or fatigue cracking [16,17].

The proposed theoretical model is capable to describe the fatigue crack propagation in marble slabs due to cyclic thermal loading. The model describes the progressive deflection (bowing) of marble slabs used in building façades. The bowing is caused by a series of concurring factors [5,13], including:

(i) Thermal gradient along the thickness of the slab due to different values of temperatures between the outer side (related to external environment conditions) and the inner side.
(ii) Such a thermal gradient causes a thermal stress field acting in the slab.
(iii) In the presence of microcracks in the slab, the nearby stress level is intensified.
(iv) Cyclic values of temperature on the outer (and inner) side(s) of the slab produce a cyclic thermal field in the slab and in turn crack propagation.
(v) Crack propagation determines a deflection of the slab, which is the sum of an elastic bulk deformation and a deformation due to the presence of microcracks. The latter increases with time as microcracks propagate through the slab thickness under cyclic thermal loading.

Thermal stress field is calculated according to Fourier's law of heat conduction, while bowing is determined by applying classical concepts of linear elastic fracture mechanics. In this way an estimation of the material durability in the presence of microcracks under thermal cyclic actions is made possible. The time histories of the temperature distribution on the outer and inner sides of the slab, and the mechanical boundary conditions at the slab ends are the main input data required by the model. The deflection of the slab, sum of the elastic bulk deformation and the deformation due to the presence of cracks, is then computed on the basis of the thermal stress field.

2.1. Temperature field in the slab

A 1D heat conduction analysis is performed in order to obtain the temperature distribution along the thickness of the slab under prescribed initial and boundary conditions. The slab has a thickness h and it is submitted to given values of temperature at the inner side ($x = 0$, Fig. 1) and at the outer side ($x = h$, Fig. 1). Assuming an uncoupling condition between temperature and stress/strain, heat conduction through the slab is described by the well-known Fourier's law:

$$\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2} \tag{1}$$

where $T(x,t)$ is the temperature function of time t and space x, and k is the diffusion coefficient (for marble $k = 0.0118\ \mathrm{cm^2/s}$, e.g. see Ref. [18]).

In order to solve Eq. (1), the following conditions have to be known:

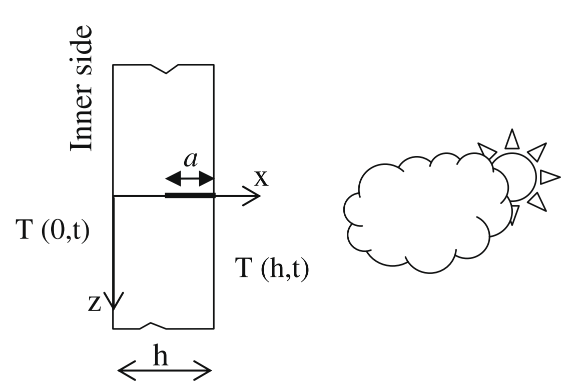

Fig. 1. Marble slab with a single external edge crack of length a under thermal loading.