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A fractal model of the porous microstructure of earth-based materials

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Abstract

Soils with a clay component have been widely used as construction materials since the earliest times and now it continue to be used in some developing countries and in the so-called bioarchitecture. This paper presents the experimental porosimetric data obtained for 13 different earth-based materials using the mercury intrusion technique. The porous microstructure is geometrically described by means of two models based on fractal geometry, the Menger' sponge and an original P' sponge proposed here. The P' sponge is constructed using an aggregation process of granular components, resembling the formation and texture of soil. The fractal dimension of the experimental data set is determined following Pfeifer and Avnir's approach. Generally the fractal range spans two to three orders of magnitude, from 20 to 0.02 μm. The results show the P' sponge to be more representative than the Menger' sponge.

Keywords: Earth; Fractal geometry; Mercury intrusion porosimetry; Models of porosity; Stabilized earth

1. Introduction

Archaeological evidence indicates that the use of raw earth as a building material dates back to around the 10th Millennium B.C. [1,2]. This material came to be widely used as it was available virtually anywhere, inexpensive and construction techniques were simple, though its performance cannot compare with other building materials such as stone and terracotta, especially in terms of resistance to water damage. Even when stone started to be used as a building material and as stoneworking technology developed over the centuries, earth continued to be employed as mud mortar, sealer or plaster in stonework or in the construction of the humblest dwellings (mixed with straw for hut roofs, beaten floors, adobe, pisé). Over time, with the development of alternative materials, such as hydraulic cements and bricks, the use of raw earth has gradually diminished, but earth-based architecture has survived in the world's less prosperous countries and is enjoying revival in the more technologically advanced nations owing to the recent move towards the so-called bioarchitecture [3,4].

It is well known that a knowledge of the pore system of construction materials provides an important contribution to an understanding and regulation of their technical properties. For example, a knowledge of void structure is basic to an understanding of compressive strength, thermal conductivity and the mechanisms of deterioration processes. However, pore geometry is not easily described. Pore size, shape, surface area and connectivity all exhibit varying levels of complexity. Simplification has traditionally been introduced to enable these geometric problems to be described in ''Euclidean'' terms (simple lines, planes, volumes of integer dimensions 1, 2, 3) but the inadequacies of this approach are notorious. There is strong interest surrounding the possibility of describing lifelike geometric structures (often irregular, jagged, fragmented) according to a different paradigm. Geometry is now able to sum up the complexity of real objects (biological and inorganic) and has brought into consideration forms which in the geometry of the early 1900s were often defined as

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''monsters''. One of these is Peano's curve. This curve develop in a plane continuously occupying it and it final dimension is not 1 but 2. One of the new conceptual instruments at the disposition of this research is ''Fractal Geometry'', which was developed during the last century but which is especially linked to its recognition and formalisation in the 1970s by Mandelbrot [5]. In its simplest version a geometric structure is fractal when a pattern repeats itself identically to any observational scale. Operationally this is feasible through a transformation which forecasts, in general, a succession of traslations, rotations and above all contractions. The fractal dimension, D_f, is the parameter which currently represents the geometric fractal structure (but it is not the only one, moreover it can be expressed in a number of ways).

In this research it is tested the ''fractality'' of materials made from a earth as received and the series of modified systems with the addition, aimed for improving technological performance [6], of sand, hydraulic cements, lime and organic polymers, from a porous microstructure data, obtained by measures of mercury intrusion. Using the fractal geometry it is also possible to elaborate on a model of the materials microstructure. The modelling of the porous geometry of the materials follows on from a long tradition of study. Modelling void geometry has been extensively researched over the years and these studies are not merely theoretical but aim at developing tools which enable to simulate transport phenomena for instance [7,8]. Some traditionally used relations, such as the Kozeny–Carman equation for predicting permeability, and Archie's law for describing electrical resistivity can also be developed representing pores as fractals [9–12].

2. Fractal models of the porosity of materials

Feasible models can be classified in a variety of groups: periodical (with traslational symmetry; are the simplest and most traditional, characterised by an ordered repetition in the space of a definite geometric element), fractal (with dilational symmetry; such as when the characteristics of the geometric structure repeat themselves identically, self similarity, to different scales), random (typically represented by percolation networks, which can in turn present identifiable fractal features). Actual examples are often so complex that for the realisation of a model different types are considered jointly, each of which is valid over a certain range of dimensions [8].

One of the most noted three dimensional fractals to be considered in this research is the ''sponge'' proposed by Menger (later referred to as M's, Fig. 1). It is an intrinsically porous structure. Operationally it is created by splitting in three equal parts each of the edges of the original solid cube, creating 27 sub-cubes. Seven of these will be eliminated, that of the nucleus and the central one from each face of the cube; on the 20 remaining sub-cubes the same process of subdivision will repeat. The procedure is therefore that of obtaining empty sub-cubes (pores) of ever decreasing dimensions. In modelling terms the development of M's sponge is within and limited dimensionally to the measurements of the edge of the larger and smaller cubes. Subsequently we will indicate as M's also those porous solids with a number of eliminated sub-cubes at every stage of the development different to 7 [13].

This study proposes an original model (henceforth referred to as P's) with a fractal component constructed not through a progressive subtraction from an initial solid block but through a process of aggregation of granular components. In fact, soils are formed by the continuous addition of solid particles of diverse mineral composition and above all characterised by a granulometry which extends over 5 or 6 orders, from the dimensions of nanometer to those of centimeter. The morphology of these particles varies from the rounded and sub-angular of the larger types, sand and silt (left out the case in which a significant percentage of mica, a lamellar like mineral, is present), to the flatter form which characterises the clay component [14–16]. The accumulation of these granules creates a complementary system of voids which connect and communicate with each other. The model proposed herein has been developed on two levels: a basic unit, typically 2 mm in size, which contains the porous fractal microstructure, and an aggregation of these units which do not have a fractal imprint.

As we can see in Fig. 2, the basic unit has a dodecagonal cross-section (of side l_1) of a maximum size of 2 mm in which is inscribed 19 dodecagons of side l_2 ($l_1/l_2 = 4.73$). A system conceived like this creates a geometry made up of triangles and stars. These stars become partially obstructed by a (solid) central square section. In this way all of the spaces now have triangular transverse sections. To simplify the formation of this spatial geometry it is assumed that this structure develops regularly to a height of 2 mm. Fig. 2 shows an example in which 7 dodecagons of 19 are scaled; the corresponding fractal dimension is 2.26 (other cases: 10 dodecagons/19, $D_f = 2.48$; 11/19 $D_f = 2.55$; 12/19 $D_f = 2.60$).

The transverse dimension of the pores, in terms of the radius (according to Washburn's equation used in the mercury intrusion technique) is estimated as being the same as the radius of the circumference that has an area equal to the triangular one.

For an evaluation of the fractal dimension from a porosimetric distributions obtained experimentally through mercury intrusion there are in scientific literature many different procedures. Fractal dimension value calculated from the different methodologies are generally consistent between them. The first approach developed, in 1983, is that of Pfeifer and Avnir [17,18]. This is used also in this work, because it is one of the most practical and today largely used. It is based on the relation: $-\frac{dV}{dr} \propto r^{2-D_f}$, where V is the volume which corresponds to the class with the radius r. The D_f is therefore generally estimated by transferring the experimental data in a bilogarithmic diagram (bi-log) and evaluating the slope of the straight line of the best correlation.