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Moment/thrust interaction diagrams for reinforced masonry sections

Y. Chen [a], A.F. Ashour [b,*], S.W. Garrity [c]

[a] Department of Aerospace Engineering, University of Bristol, Bristol BS8 1TR, United Kingdom
[b] School of Engineering, Design and Technology, University of Bradford, Bradford, West Yorkshire BD7 1DP, United Kingdom
[c] Garrity Associates, c/o School of Engineering, Design and Technology, University of Bradford, BD7 1DP, United Kingdom

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Abstract

This paper presents a numerical analysis for estimating moment/thrust interaction diagrams for reinforced masonry sections. Realistic constitutive models are used for the masonry and steel reinforcement. Considering compatibility of strains and equilibrium of forces on reinforced masonry sections, the moment–curvature relationships for different axial thrusts are first produced. The moment–thrust interaction diagrams are then developed. Comparisons between the behaviour predicted using the current method and experiments conducted on reinforced and unreinforced masonry short columns are presented to examine the validity of the developed method. Finally, a parametric study of the material properties is undertaken.

Keywords: Reinforced masonry; Interaction diagrams; Columns, Capacity

1. Introduction

Most masonry structures, such as walls and columns, are loaded with a combination of compression and bending. The bending action can be created by vertical load eccentricity or the application of lateral loading; it can cause extensive cracks as masonry is characterised by a very low tensile strength.

Over the years, reinforced masonry has been used increasingly in construction. Reinforcing masonry improves its bending resistance, increases its vertical load bearing capacity and changes the material behaviour from brittle to ductile. The main purpose of the described research in this paper is to investigate the flexural behaviour of reinforced masonry sections. The moment–curvature and moment–thrust interaction diagrams are numerically developed based on realistic constitutive relationships of masonry and steel reinforcement. The validity of the proposed numerical technique is examined by comparing the predicted flexural behaviour of masonry sections with those obtained from physical testing. A parametric study of the material properties is also conducted and presented.

2. Constitutive material models

2.1. Masonry in compression

The resistance of masonry to the compressive loading has been the subject of systematic investigation over a very considerable period [6]. A number of tests had been carried out using different combinations of bricks and mortar to determine the compressive stress–strain relationship [9,3]. It was found that the relationship could be represented by a smooth curve with decreasing elastic modulus up to the ultimate stress beyond which there is a strain softening. A wide range of ultimate strains was encountered for different types of brickwork. The size of masonry test specimens for determining stress–strain characteristics was also found to have some effect on ultimate strain at failure; large specimens were weaker, failed at a lower ultimate strain and exhibited a smaller range of strains than smaller specimens [7].

* Corresponding author. Tel.: +44 1274233870; fax: +44 1274234111.
E-mail address: afashour@bradford.ac.uk (A.F. Ashour).

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The constitutive model for masonry in compression adopted by the authors is that determined by Taylor and Mallinder [13] based on the experimental data of Hudson et al. [18], Hodgkinson and Davies [7], Towler and Sawko [14] and Taylor and Mallinder [12]. This model is a parabolic curve as shown in Fig. 1 and can be represented by:

$$\sigma = \left[\frac{k}{k-1} \frac{\varepsilon}{\varepsilon_m} - \frac{1}{k-1} \left(\frac{\varepsilon}{\varepsilon_m} \right)^k \right] \sigma_m \tag{1}$$

where σ and ε are the instantaneous compressive stress and strain, respectively; σ_m represents the masonry compressive strength and ε_m is the corresponding strain; k is a constant ($k > 0$) for a given structural masonry material; $\varepsilon = \lambda\varepsilon_m$ denotes the maximum strain at which crushing occurs as given in Fig. 1. In addition, the initial tangent modulus of the stress–strain curve, E_0, can be defined by $E_0 = \left(\frac{k}{k-1}\right)\frac{\sigma_m}{\varepsilon_m}$.

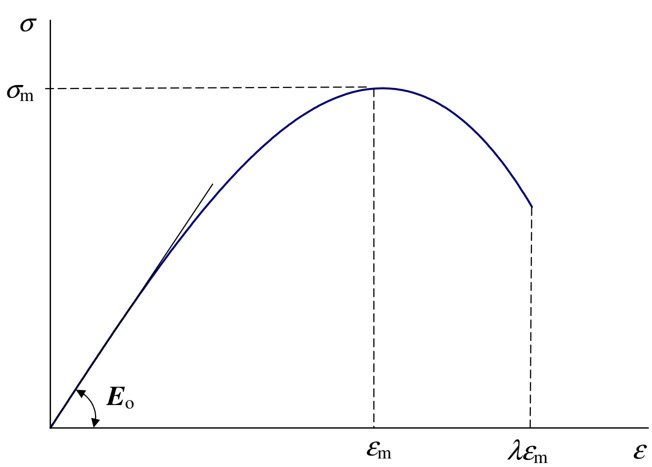

Fig. 1. Stress–strain relationship of masonry in compression [13].

2.2. Masonry in tension

The tensile strength of masonry is very small and it is usually ignored in the design or strength assessment of masonry structures. However, in order to overcome the convergence problems encountered in the numerical analysis presented in this paper, masonry tensile strength is necessary to be considered.

Very little experimental work seems to have been conducted to develop the stress–strain relationship of masonry in tension. From the limited amount of tensile tests on small masonry specimens [17,16], it was found that the masonry behaved nearly linear before the tensile strength was reached. Then the masonry showed a non-linear behaviour usually known as tension softening, in which the tensile stress gradually decreased to zero rather than immediately dropped to zero. A modified form of the stress strain relationship developed by Reinhardt et al. [10] as illustrated in Fig. 2 is adopted in this study.

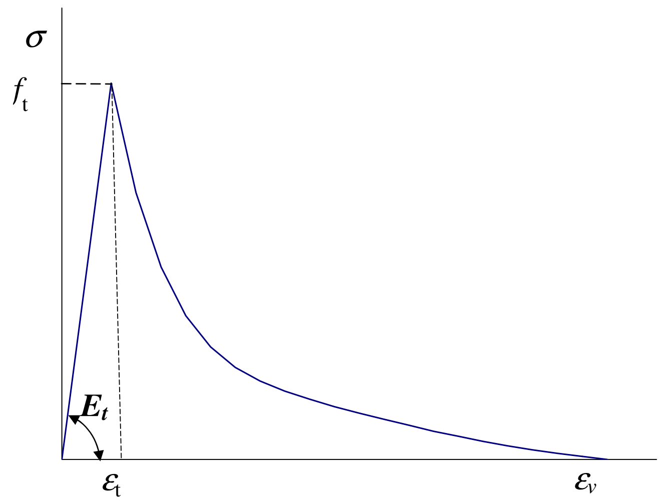

Fig. 2. Stress–strain relationship of masonry in tension.

The model of masonry in tension can be given in the following analytical form:

$$\sigma = \begin{cases} E_t\varepsilon & \varepsilon \leqslant \varepsilon_t \\ K_r f_t & \varepsilon > \varepsilon_t \end{cases} \tag{2}$$

where $K_r = [1 + (c_1 \frac{\varepsilon-\varepsilon_t}{\varepsilon_v})^3] \exp(-c_2 \frac{\varepsilon-\varepsilon_t}{\varepsilon_v}) - \frac{\varepsilon-\varepsilon_t}{\varepsilon_v}(1+c_1^3)\exp(-c_2)$, E_t is the elastic modulus in tension that, for simplicity, is assumed to be equal to E_0 in compression as presented earlier; ε_t ($=f_t/E_t$) is the strain corresponding to the tensile strength f_t of masonry; $c_1 = 3$; $c_2 = 6.93$ and ε_v is the strain corresponding to zero stress. A small value of ε_v (=0.0003) is used here to reflect the brittle nature of masonry in tension. K_r gives the rate of tension softening of the test specimen. A smooth, slow rate of tension softening is assumed here in order to overcome any numerical problems and achieve numerical convergence of the analysis.

2.3. Steel reinforcement

The constitutive model of steel reinforcing bars given in BS 5628: part 2 the "Code of practice for use of masonry" [2] is adopted, as illustrated in Fig. 3. It can be written in the analytical form given below. For steel reinforcement in tension:

$$\sigma_s = \begin{cases} E_s\varepsilon_s & 0.8f_y/E_s \geqslant \varepsilon_s \geqslant 0 \\ 0.17E_s\varepsilon_s + 0.664f_y & 1.976f_y/E_s > \varepsilon_s > 0.8f_y/E_s \\ f_y & \varepsilon_s \geqslant 1.976f_y/E_s \end{cases} \tag{3}$$

For steel reinforcement in compression:

$$\sigma_s = \begin{cases} E_s\varepsilon_s & 0.8f_y/E_s \geqslant \varepsilon_s \geqslant 0 \\ 0.17E_s\varepsilon_s + 0.664f_y & 0.976f_y/E_s > \varepsilon_s > 0.8f_y/E_s \\ 0.83f_y & \varepsilon_s \geqslant 0.976f_y/E_s \end{cases} \tag{4}$$

where σ_s and ε_s denote the stress and strain in steel reinforcement, respectively; f_y is the tensile yield strength and