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Nonlinear behavior of slender RC columns (1). Numerical formulation

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Abstract

This paper introduces an analytical model which can simulate the nonlinear behavior of slender reinforced concrete (RC) columns. The layer approach is adopted to determine the equilibrium conditions in a section and to consider the different material properties across the sectional depth effectively. The material nonlinearity including the cracking of concrete is taken into consideration, and geometric nonlinearity due to the *P*–*Δ* effect is taken into account by using the initial stress matrix. In advance, the creep deformation of concrete is described in accordance with a first-order algorithm based on the expansion of a degenerate kernel of the compliance function. To verify the analytical results, correlation studies with previous numerical results and experimental data are conducted, and numerous parameter studies are followed to discuss the structural responses of slender RC columns according to the changes in design variables. Finally, the necessity for a rigorous nonlinear analysis is emphasized for more accurate prediction of the ultimate resisting capacity of slender RC columns.

Keywords: RC column; Ultimate resisting capacity; Slenderness ratio; *P*–*Δ* effect; Creep

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1. Introduction

A reinforced concrete (RC) column, which is a primary structural member, is subjected to the axial force and bending moment which may be due to end restraint arising from the monolithic placement of floor beams and columns or due to eccentricity from imperfect alignment. Due to the combination of axial force and bending moment, the column section must be designed to ensure that the acting forces in a member exist inside the *P*–*M* interaction diagram representing the resisting capacity of the column. Recently, because of architectural aesthetics and efficiency in use of space, relatively slender columns have frequently been used in many building structures, either throughout an entire building or in some parts of a structure, e.g., the exterior of buildings and the interior of lobbies. Moreover, the use of high strength steel and concrete has led to an increased use of slender members. However, as slender RC columns may fail due to not only material failure in a section but also instability of a structure, they require more rigorous numerical analyses which consider secondary effects such as the *P*–*Δ* effect and creep deformation of concrete in order to reserve their strength and serviceability.

There has been a lot of research on the behavior and design of slender RC columns. On the basis of the force equilibrium equation and the strain compatibility condition at a section, Bazant et al. [3,5] analytically calculated the resisting capacity of slender RC columns by assuming a deflection curve with a sinusoidal function. Material nonlinearity of steel and concrete was taken into

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account and an excellent discussion of slenderness effects on interaction diagrams was provided. Kim et al. [10,11] introduced a numerical method considering material and geometric nonlinearities by using the layer model and carried out an experimental study to verify the exactness of the algorithm they developed. Drysdale and Huggins [8] conducted experimental and analytical studies for both short-term and long-term behaviors of RC columns with relatively high slenderness ratio and discussed decrease of the ultimate resisting capacity due to the *P–Δ* effect and creep deformation of concrete. Recently, Yalcin and Saatcioglu [21] developed an analytical model that considers the influence of anchorage slip and plastic hinge length on the nonlinear behavior of RC columns.

In this paper, an analytical model to predict the resisting capacity of slender RC columns is proposed. The layer approach is adopted to simulate the different material properties across the sectional depth. Material nonlinearity due to the cracking of concrete and yielding of reinforcing bars, and geometric nonlinearity due to the *P–Δ* effect are taken into account. Concrete creep is evaluated by the first-order algorithm based on the expansion of the compliance function [9], and the aging effect of concrete properties is included in the evaluation. The validity of the numerical model proposed in this paper is established by comparing the analytical predictions with results from previous analytical studies [5,10], and numerical analyses for slender RC columns are conducted. On the basis of the numerical results obtained, the necessity of rigorous nonlinear analysis is emphasized for more accurate prediction of the ultimate resisting capacity of slender RC columns.

2. Structural behavior of slender RC columns

Generally, the ultimate compressive force P_0 and the ultimate bending moment M_0 for an RC column section are related to each other by means of an interaction diagram (*P–M* interaction diagram). In the absence of second-order effects (*P–Δ* effect), as in very short columns, the cross-section would undergo proportional loading until reaching the material strength at point A of the cross-section interaction diagram (see Fig. 1). Slender columns, however, will follow the loading path up to point B where the material strength is reached. Point B is on the cross-section interaction diagram but is at a smaller axial load, P_{so}, than it would be if L/r were actually zero. Unlike a steel column, a concrete column accompanies relatively large time-dependent deformation, such as creep. This time-dependent deformation gradually increases the lateral deflection caused by the initial eccentricity *e* and the *P–Δ* effect, and finally decreases the ultimate resisting capacity and serviceability of slender RC columns.

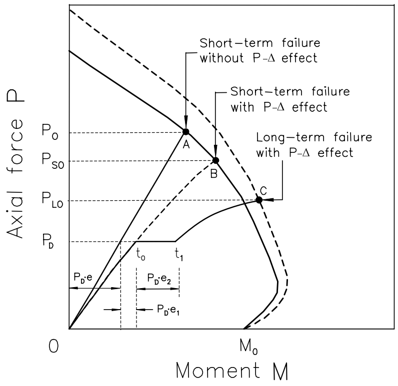

Fig. 1. Behavior of slender RC column.

In the case of an RC column with a relatively large slenderness ratio, instability failure, which means a failure before reaching the *P–M* interaction diagram of an RC cross-section (the solid line envelope in Fig. 1), may occur. A typical description for the long-term failure is illustrated in Fig. 1. When an axial load P_D with initial eccentricity *e* acts on a slender RC column at $t = t_0$, the instantaneous lateral deflection will be e_1 due to second-order effects (*P–Δ* effect). Moreover, the creep deformation during $t_0 \sim t_1$ will increase the lateral deflection in spite of the absence of additional axial load. If an additional live load is applied at $t = t_1$, its increment terminates at point C, located outside the cross-section interaction diagram, because the strength for a live load applied after a period of creep under constant load P_D is usually higher than the short-term strength. The larger the column slenderness ratio, the greater is the reduction in the axial force resistance.

For not too slender columns, the failure occurs at points rather close to the material strength. For very slender columns, on the other hand, the failure occurs well within the cross-section interaction diagram because of a pronounced second-order effect. Referring to the structural behavior of RC columns, material nonlinearities of steel and concrete and time-dependent deformation of concrete are taken into account for more exact estimation of the ultimate resisting capacity of RC columns. Moreover, it needs to be assumed that the failure of RC columns occurs due to material failure only because the slenderness ratio in most RC columns designed in practice is smaller than the critical ratio that causes instability failure, even if the slenderness ratio ranges to a large value.

3. Modeling of material properties

3.1. Concrete

Based on the principle of superposition, total uniaxial concrete strain $\varepsilon_c(t)$ at any time *t* is assumed to be