Available online at www.sciencedirect.com
ScienceDirect
Construction and Building Materials 21 (2007) 1390–1398
Construction and Building MATERIALS
www.elsevier.com/locate/conbuildmat

Confidence intervals of prediction and synthesis of prediction estimates for deformability of expanded polystyrene in long-term compression

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Received 13 January 2006; received in revised form 17 July 2006; accepted 1 August 2006
Available online 12 September 2006

Abstract

The paper presents an experimental and numerical study of expanded polystyrene (EPS) with density ranging from 14.5 to 33 kg/m^3, which was subjected to long-term compression of $\sigma_c = 0.35\sigma_{10\%}$ to verify the suggested methods of predicting compressive strain development in EPS products. The total time of testing was 608 days. Interval prediction of creep strain development for the period of 50 years was made by extrapolation based on power and exponential regression equations applied to approximate creep formation. These equations were reduced to a linear form by using logarithms. An additional factor *K*, depending on the number of retrospective test results and non-dimensional intervals in the range of prediction is used for a linear trend to correct for the expansion of confidence interval due to discounting of prediction information. Predictions obtained by using power and exponential equations were synthesised and the results were discussed.

Keywords: Expanded polystyrene (EPS); Long-term compression; Creep; Interval prediction; Synthesised prediction

1. Introduction

Products of expanded polystyrene (EPS) are extensively used in several applications, especially as geofoam, where its mechanical strength is very important [1–3]. Therefore, the time-dependent mechanical (stress–strain) behavior of these products has been a topic of significant interest for many years [2–6]. Many authors propose the simplified power Findley equation for making long-term prediction of the creep strain development in EPS [3–5].

Making long-term prediction of the creep strain development in expanded polystyrene (EPS) requires a comparison of models based on extrapolation. However, the models possessing similar statistical characteristics may be obtained, making it difficult to give preference to one of them. It may be assumed that each model describes only a particular aspect of the creep strain presented in the analysed time series. Therefore, the use of several models allows us to predict and describe strain formation more accurately from various perspectives. In this process, a so-called synthesis of prediction estimates can be used to obtain a synthesised prediction. Consistency of predictions may be determined by comparing their confidence intervals (i.e. the deviation of the predicted values from the actual values).

A comparison between Findley equation results and a simpler power-law model is given in [5]. The publication [7] is intended for comparison of prediction results when extrapolation is based on power and exponential equations. These investigations based on power model have been continued and extended by an attempt of interval prediction in [8].

In establishing prognostic creep values by extrapolation, the determination of confidence intervals of prediction seems to be more interesting and important than the extrapolation itself, which is actually a standard technique. Confidence intervals take into account the uncertainty caused by a limited number of observations and the resulting inaccuracy of the estimates obtained for the empirical curve parameters.

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0950-0618/$ - see front matter
doi:10.1016/j.conbuildmat.2006.08.001

The aim of the present investigation is the determination of confidence intervals and synthesis of the prediction estimates of expanded polystyrene creep based on power as well as exponential models and direct long-term experiments.

2. Testing methods and data processing

Expanded polystyrene boards of the types EPS 60, EPS 100, EPS 120, EPS 150, EPS 200 [9] with the density of 14.5–33 kg/m^3 made by foaming beads (solid granules of 0.9–2.5 mm in diameter supplied by the companies "STYROCHEM" and "BASF") were tested.

Creep strain curves were obtained when the specimen cubes with 50 mm-edge were subjected to long-term compression on special stands [10], providing the development of a fixed stress $\sigma_c = 0.35\sigma_{10\%}$ for 608 days, with $\sigma_{10\%}$ being the stress corresponding to 10% specimen deformation. The compressive stress was assumed to be perpendicular to the plane of the plate from which the specimens were cut. The error of long-term compressive stress did not exceed ±1%, while the error of creep strain measurement was about ±0.005 mm. Each of six tests was made on three specimens of the same density. The procedures of loading the samples and taking the reading of dial gauges were carried out according to the specifications [10]. Thus, each specimen was loaded during (10 ± 5) s. The starting point of the creep was assumed to be the indication of strain after (60 ± 5) s from the beginning of loading.

Long-term tests were made at a temperature of 23 ± 2 °C and air humidity of 50 ± 5%.

To describe the curves of expanded polystyrene creep obtained in testing, power (1) and exponential (2) equations were used. First of them is recommended for thermal insulating products by [10], the second one had been checked for this purpose in [8]

$$\bar{\varepsilon}_c(t) = b_0 t^{b_1}, \tag{1}$$

$$\bar{\varepsilon}_c(t) = b_0(1 - e^{-b_1 t^{b_2}}), \tag{2}$$

where $\bar{\varepsilon}_c(t)$ is the experimental creep strain value, %, at the moment t; b_0, b_1, b_2 are constant factors depending on material characteristics, temperature and stress, computed on the basis of experimental data; t is time (h) in Eq. (1), and days in Eq. (2).

The constant factors of the empirical relationships (1) and (2) – 'creep strain – duration of the compressive stress σ'_c were determined by technique of least squares [11]. Standard deviation S_r (i.e. an absolute value of mean test data deviation from the empirical regression line, which is invariable for all sections of this line) was assumed to be a measure of the test data spread about the regression lines (1) and (2). It was found from the formula:

$$S_r = \sqrt{\frac{\sum_{i=1}^{n}[\varepsilon_c(t) - \bar{\varepsilon}_c(t)]^2}{n-m}}, \tag{3}$$

where $\varepsilon_c(t)$ is an experimental creep value at the moment t; $\bar{\varepsilon}_c(t)$ is the same based on empirical relationships (1) and (2); n is the number of test results; m is the number of parameters of empirical equation ($m = 2$ or $m = 3$).

The determination factor $\eta^2_{\varepsilon_c \cdot t}$ (the square of correlation ratio) [11,12] showing the dependence of the creep strain ε_c variation on the variation of t (if $\eta^2_{\varepsilon_c \cdot t} \approx 1$, then the regression curve passes through all test results) was assumed to be an indicator of the interrelationship of the variables ε_c and t in the non-linear regressions (1) and (2).

In general terms, the confidence interval for the trend of creep strain prediction can be evaluated in the following way:

$$\bar{\varepsilon}_c(t) \pm t_{\alpha,f} \cdot S_r, \tag{4}$$

where $\bar{\varepsilon}_c(t)$ is a point prediction value for creep strain (according to empirical equations, e.g. (1) or (2)); $t_{\alpha,f}$ is the value of the Student's t-statistics with the degrees of freedom $f = n - m$ and the significance level α.

For the time ($t_L = n + L$) the expression (4) evaluates the confidence interval for the trend extended by L units of time. In this case, the confidence interval of prediction should take into account not only the uncertainty associated with the trend itself but possible deviations of this trend as well.

Let the respective standard (prediction) deviation be denoted by S_{pr}, then, the confidence interval of the prediction will be expressed as follows:

$$\bar{\varepsilon}_c t_L \pm t_{\alpha,f} \cdot S_{pr}. \tag{5}$$

The empirical relationships (1) and (2) can be made linear by using logarithms. Then, the standard deviation of a prediction estimate for a linear trend of extrapolation may be expressed in the following way [13,14]:

$$S_{pr} = S_r \cdot \sqrt{\frac{n+1}{n} + \frac{(t_L - \bar{t})^2}{\sum_{t=1}^{n}(t - \bar{t})^2}}, \tag{6}$$

where n is the number of test results; t_L is time period for which the extrapolation is applied; $\bar{t}$ is the consecutive number of the test result at the midpoint of a full series of test results; t are the consecutive numbers of test results, making a series of natural numbers.

By denoting a square root in Eq. (6) by K and basing ourselves on the proofs given in [6] of the expression (6), we can express K as follows:

$$K = \sqrt{\frac{n+1}{n} + \frac{3(n + 2L - 1)^2}{n(n^2 - 1)}}, \tag{7}$$

where L is the number of non-dimensional lead time intervals in the range of prediction.

Prediction lead time interval is numerically equal to a time step between retrospective test results. For example, when a time step between retrospective values is equal to one month, while the time of prediction $t_L = 12$ months, then, the value of $L = 12$; when $t_L = 36$ months, $L = 36$, etc.